Dielectric Constant and Loss Measurements on High-Temperature Materials

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Abstract: Measurement techniques for the frequency range 10^2 to about 2.5×10^{10} are discussed for temperatures to 1650°C. These include the use of bridges, resonant circuits, standing-wave methods, and resonant cavities. Data on crystals of Al_2O_3, Cr_2O_3, MgO, $LaAlO_3$, Y_2O_3; on multicrystalline bodies of Al_2O_3, BeO, MgO, Mg_2SiO_4, Ta_2O_5, ThO; on glass ceramics, silica glass, and BN are presented over smaller temperature and frequency ranges. Pyrolitic BN has a low loss tangent (0.0004 at 1375°C, 4.8×10^9 cps) and a low temperature coefficient of dielectric constant. Some aluminas and silicas exhibit loss tangents of ca. 0.0006 at 1000°C in the microwave region. Microwave losses are due partly to the charge transfer responsible for low-frequency conductivity and to the vibration spectra of infrared absorption. Both losses are increased by the addition of impurities.

I. Measurement Techniques

Introduction

High-temperature dielectrics generally show low loss (tan δ as low as 10^{-5}) at room temperature but exhibit loss tangents > 10 at low frequencies and high temperatures. The measurement methods must vary with loss tangent

Table 1. Sample sizes.

Temperature (^{o}C)	Shape	Diameter	Thickness	Frequency (cps)
-80^{o} to +500^{o}	disk	$1\frac{3}{4}$ to 2"	0.1 to 0.3"	$10^{2} - 10^{7}$
25^{o} to 1400^{o}	disk	$\frac{3}{4}$ to 1"	0.06 to 0.2"	$10^{2} - 10^{7}$
25^{o} to 1700^{o}	disk	$\frac{1}{2}$ to $\frac{3}{4}$"	0.2 to 0.5"	$10^{2} - 10^{8}$
600^{o} to 1200^{o}	rod	$\frac{1}{4}$ to $\frac{5}{16}$"	$\frac{3}{4}$ to 1"	$10^{3} - 10^{5}$
25^{o} to 1700^{o}	cylinder	1.000	$\frac{5}{8}$ to $\frac{7}{8}$"	$3 - 5 \times 10^{9}$
25^{o} to 800^{o}	"	1.000	$\frac{5}{8}$ to $\frac{7}{8}$"	8.5×10^{9}
25^{o} to 800^{o}	"	0.374	$\frac{3}{8}$ to $\frac{5}{8}$"	2.4×10^{10}
25^{o} to 1700^{o}	"	0.374	$\frac{1}{2}$ to $\frac{5}{8}$"	$8 - 10 \times 10^{9}$

as well as with frequency and temperature. A typical characteristic is shown in Fig. 1; various measurement zones are marked with Roman numerals corresponding to the general measurement methods listed in the figure and in Appendix A. Zone boundaries are not sharp and depend partly on operating convenience.

Typical sample sizes are indicated in Table 1. The larger samples in the lumped circuit range allow more freedom from irregularities in the edges of electrodes (silver, platinum, or carbon), diffusion, and surface conductivity. The removable sample holder and its connections prohibit accurate measurements. Dual resonant circuits with duplicate sample holders could be used to extend the frequency range, but our recent development work has indicated that parallel capacitance bridges in the range 3 to 300 Mc are feasible; they are under construction. Bridges can measure a much wider range of losses, and similar sample holders in opposite bridge arms allow cancellation of the

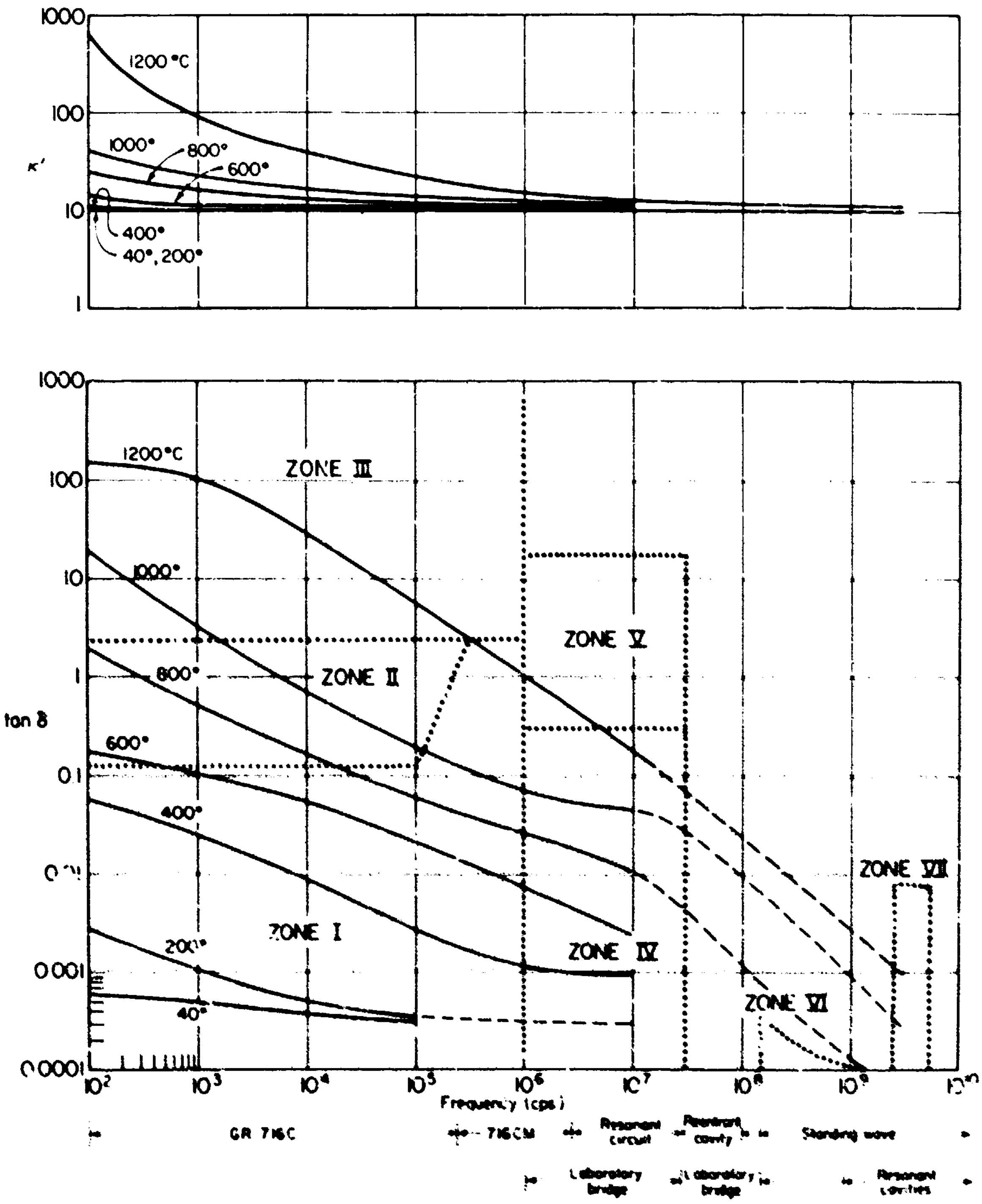
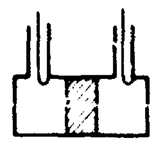
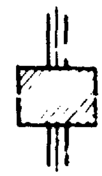
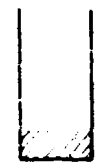
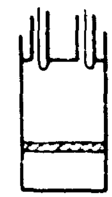


Fig. 1. Dielectric constant and loss of a magnesium oxide ceramic with measuring instruments for various frequency ranges.

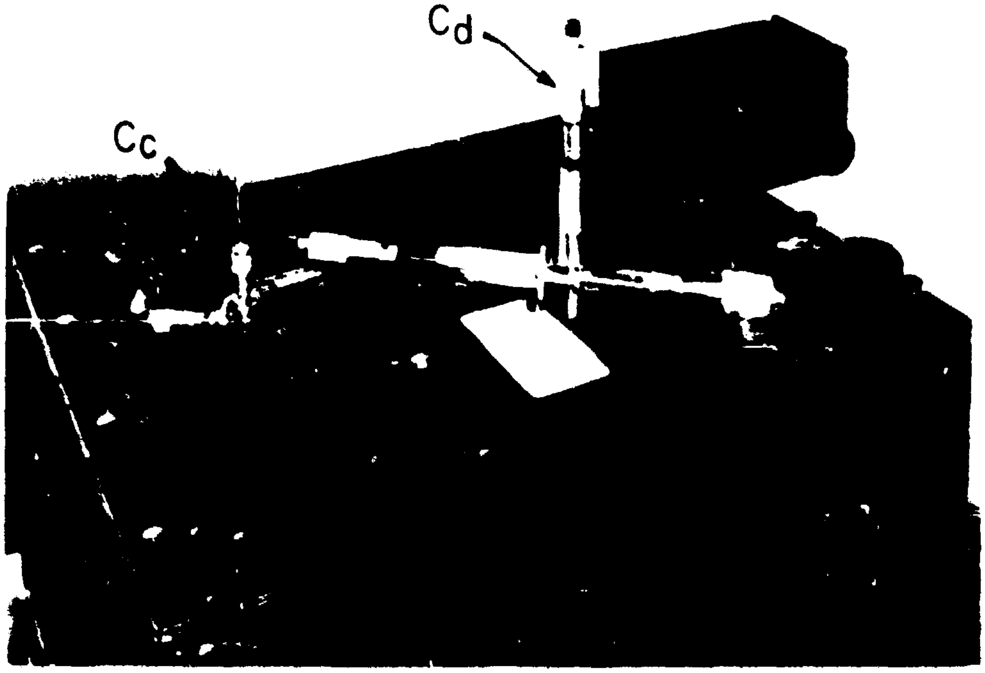
Table 2. Microwave measurements at elevated temperatures.

	Resonant cavity	Resonant dielectric-filled cavity	Standing wave	Resonant cavity
				
Wave mode	TM_{010}	TE_{111} or TM_{010}	TE_{11}	TE_{111}
Typical frequency	1 kMc	4 kMc	8.5 kMc	8.5 kMc
Sample shape	cylinder	cylinder	cylinder	disk
Sample size	1" x 7/8"	1" x 7/8"	1" x 7/8"	1" x 0.01"
Typical accuracies in percent				
in κ' at 25°	1/2	0.05	1/4	1
in κ' at 800°	2	0.1	3	1
Minimum detectable loss tangent				
silver	0.00005	0.00004	0.000001	0.002
Inconel			0.0005	0.01
platinum	0.0002	0.00005	0.0003	0.005
Upper limit on loss tangents	0.02	0.005	0.1	1

effect of series resistance on loss measurements.

Four-terminal measurements will be discussed in a later section. For most materials no suitable samples were available. A few samples of alumina were checked but showed no significant differences.

Comparisons of two-terminal measurements made on samples cooled from > 200°C in dry nitrogen with three-terminal measurements also showed no differences within limits of error.



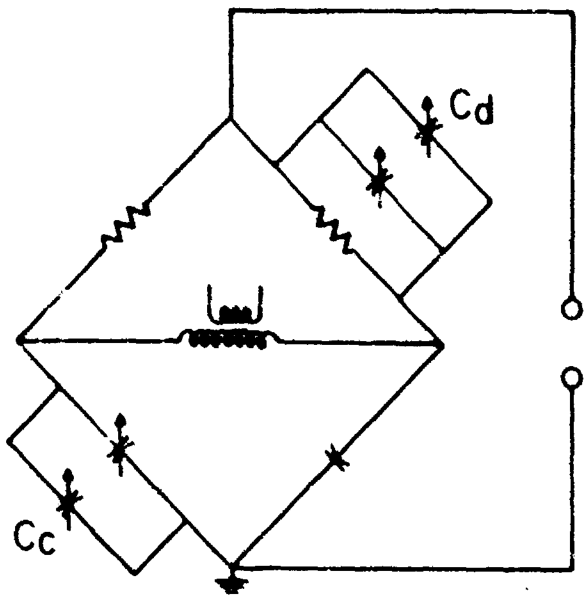
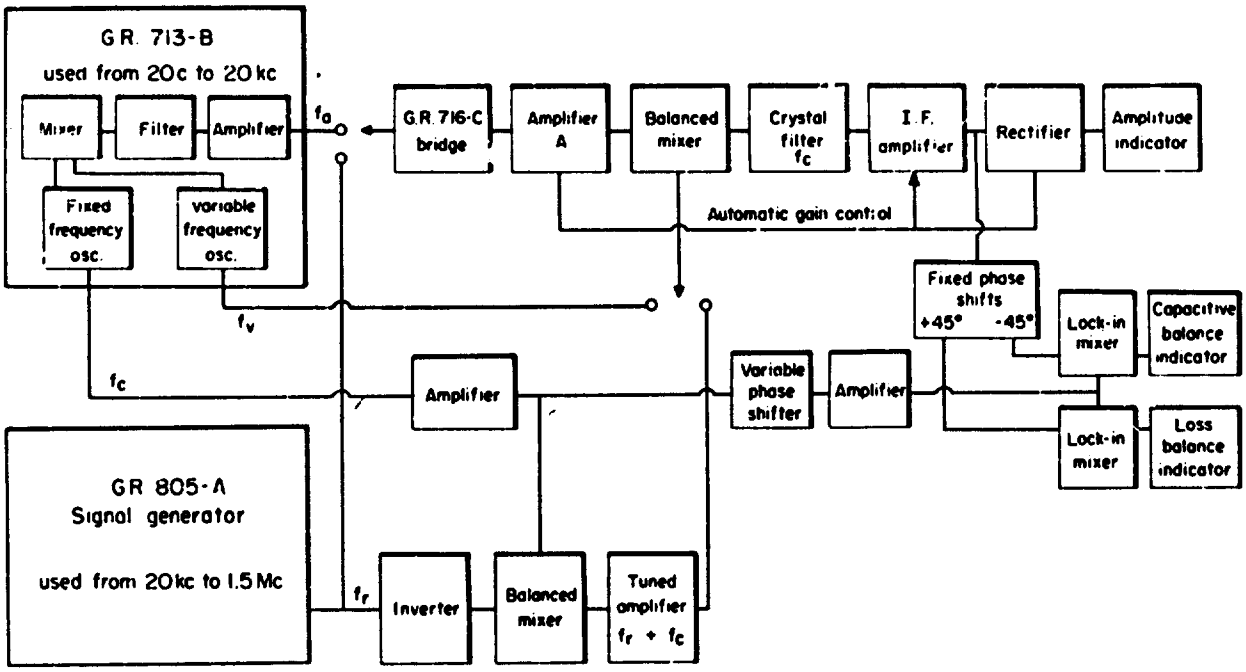


Fig. 2. Vernier capacitors (C_c and C_d) added to GR 716-C bridge. C_c is formed by micrometer shaft in proximity to the lead of precision condenser, nonlinear with a range of 0.1 pf. C_d is a linear coaxial capacitor mounted within shield box of main dissipation factor control; linear range 2 cm, 0.0477 D/cm.

In the microwave region four-measurement methods have been contemplated (Table 2). Of these only the dielectric-filled-cavity and the standing-wave methods have been extensively used. Details for these are described in later sections.



	100 c	1 kc	10 kc	100 kc	1 Mc
Minimum detectable signals in μv					
A. Using amplitude indicator:					
with amplifier A tuned	0.5	0.4	0.2	0.03	0.3
with amplifier A wide band	10	3	7	0.5	
B. Using lock-in indicator:					
with amplifier A tuned	0.5	0.2	0.1		
with amplifier A wide band	1.5	0.5	0.1		
Second harmonic rejection, db					
A. Using amplitude indicator:					
with amplifier A tuned	55	74	111		
with amplifier A wide band	15	33	65	87	
B. Using lock-in indicator:					
with amplifier A tuned	>80	>100	>115		
with amplifier A wide band	55	> 80	> 90		

Fig. 3. Bridge detector for 20 cps to 1.5 Mc.

Special Instrumentation

For the measurement of low losses, Zone I, our GR 716-C and 716-CM bridges are provided with vernier controls (Fig. 2) and laboratory-built detection equipment (Fig. 3). The latter provides minimum detectable signals in the 0.1-μvolt range with harmonic rejection in excess of 55 db. The combination results in over-all loss sensitivity of $< 2\ \mu$ radians in the range 10^2 through 10^6 cps. For typical samples (C_s ca. 30 pf) the loss sensitivity is 10^{-5} radians. The aluminum plates in the precision capacitor of the bridge have a loss, due to absorbed H_2O, of 30 μradians at 10^2 cps and 40% relative humidity. This loss was calibrated when necessary by comparison measurements with a copper-plate capacitor operating in dry nitrogen.

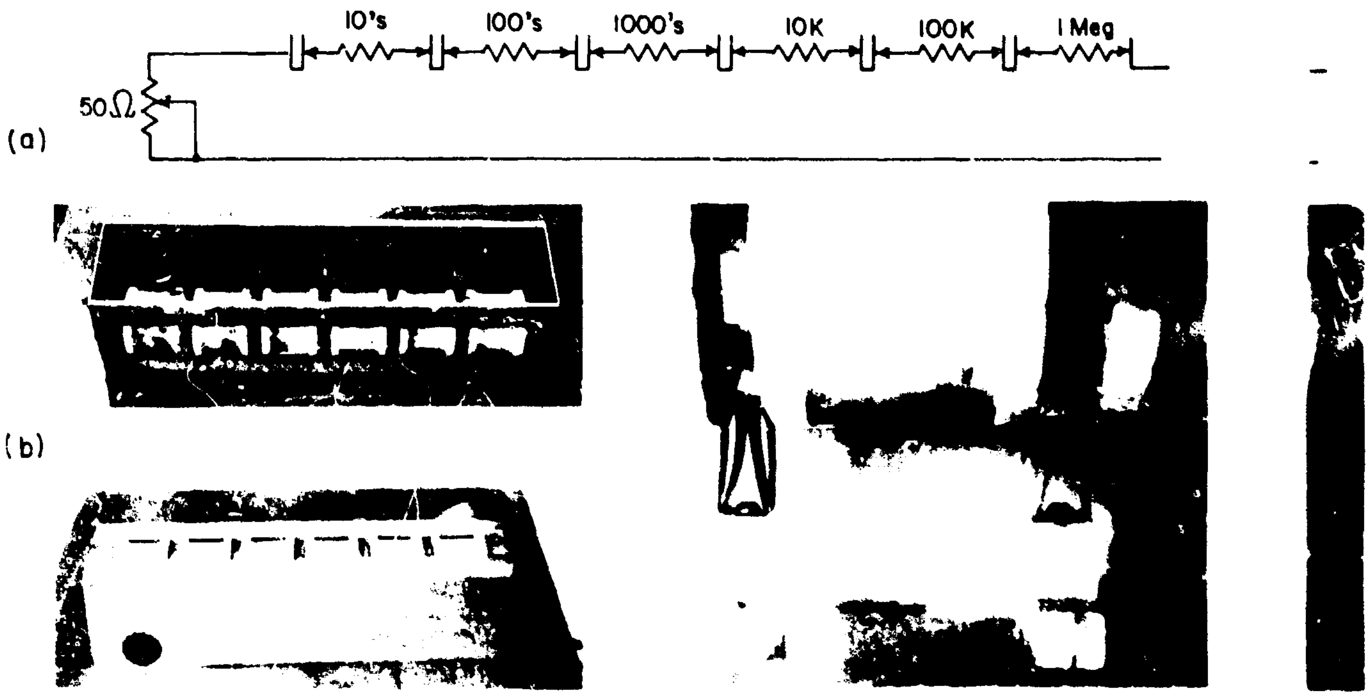


Fig. 4. Turret-type decade resistor box to 10 megohms: (a) schematic, (b) outer, inner, and contact views.

For higher losses, Zone III, a variable resistance having little change in capacitance is required to substitute for sample conductivity. Figure 4 shows the construction of a turret-type resistor in which the change in equivalent parallel capacitance is < 0.3 pf within one decade. The performance of this and other decade resistors will be analyzed in a separate report.

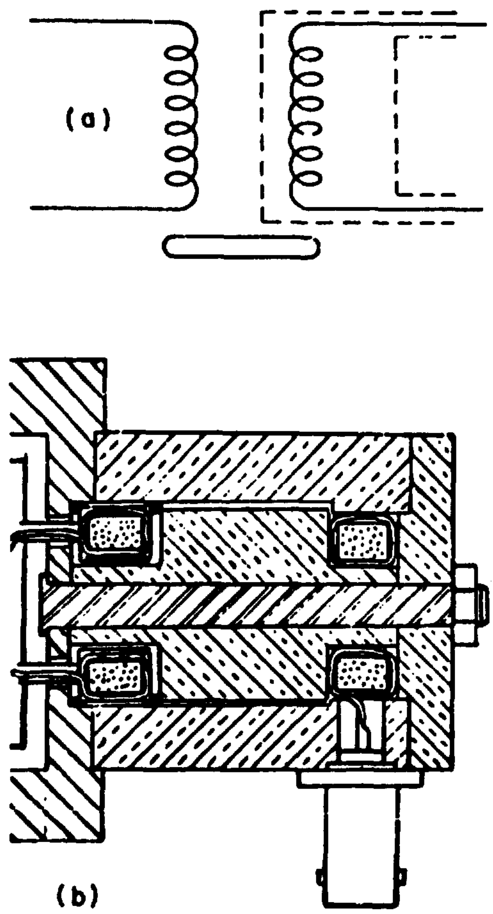


Fig. 5. High-frequency bridge transformer; (a) schematic, (b) cross section.

Conjugate Schering bridges have capacitors only for balancing elements. At high frequencies the design problems are mainly the series residuals in the capacitance balance and transformer leakage due to imperfect shielding. A series resistance of 30 μohms is necessary in a 50-pf capacitor at 100 Mc for a loss tangent of 10^{-6}. This is an impractical goal when one considers that the resistance of a 1-cm diam. copper rod is about 1 milliohm per cm length at 100 Mc. A symmetrical bridge allows the possibility of balancing out the effect of lead resistance in a second arm. This concept, combined with transformer design (Fig. 5), has been tested in a bridge for liquids 1-40 Mc but is still under development for solids.

The design of any sample holder in the lumped-circuit range is a compromise between good thermal (low-temperature gradients in the sample) and good electrical design (small resistance, inductance, capacitance, and conductance).

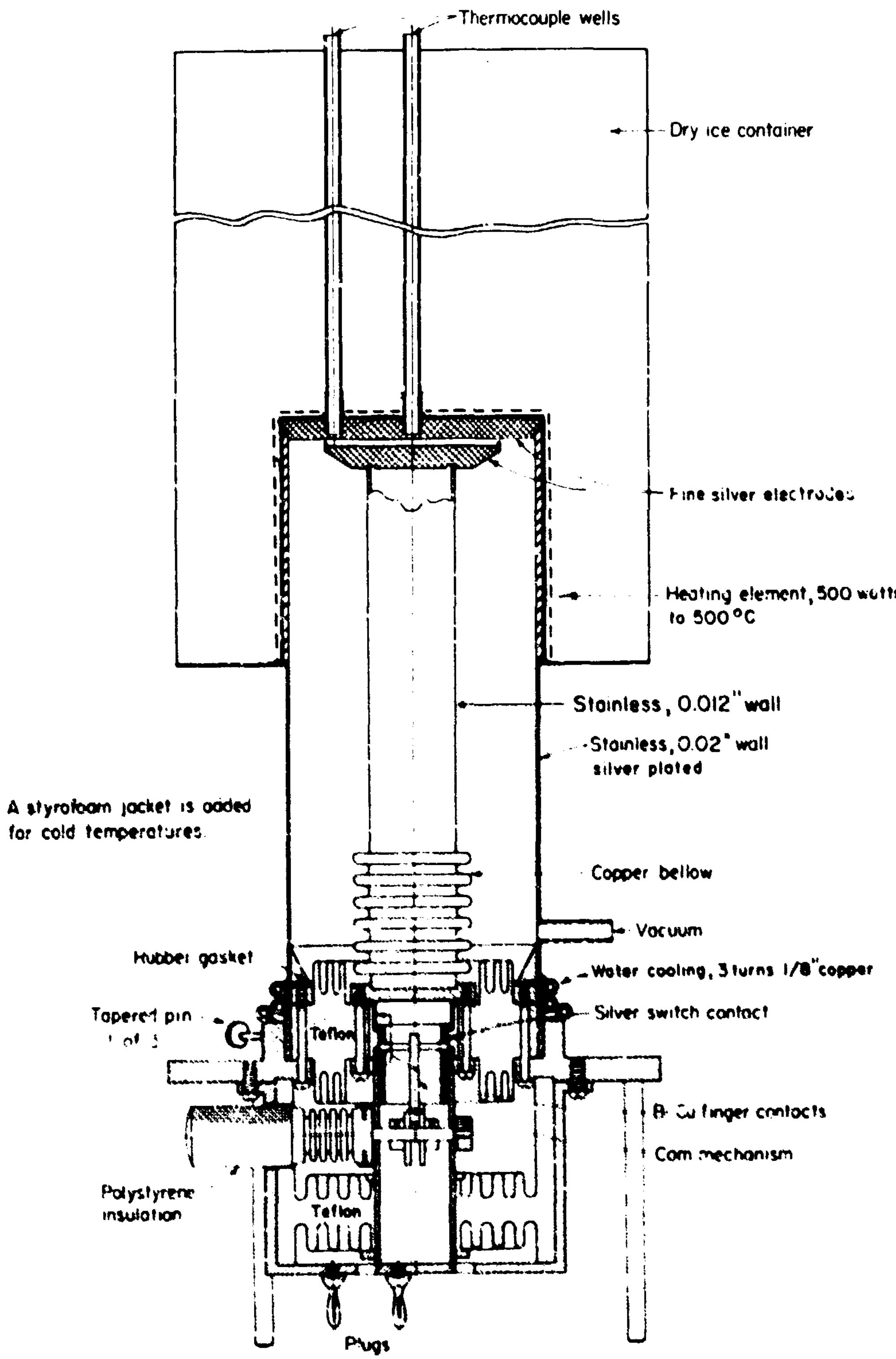


Fig. 6. Sample holder for determining changes in capacitance and loss of 1-3/4 to 2-inch disks to 500°C.

The sample holder of Fig. 6 has an inductance low enough to serve at 10 Mc without, and to 30 Mc with corrections (30-pf sample). The series resistance correction is < 0.0003 in loss tangent at 10 Mc. The temperature difference at 500°C on a 1/4-inch thick alumina is about 30°C in N_2. The built-in disconnect switch opens a gap of 0.3 pf in series with the center conductor in the "off" position; the holder capacitance is then 10 pf. In the "on" position the sample plus about 13 pf are added. The holder may be evacuated but is usually operated with a flow of dry nitrogen. A layer of aluminum foil is used to prevent the silvered sample from bonding to the electrodes. The disadvantages are that experience is required in aligning the sample and electrodes because of the flexibility of the electrode stem, and that the upper electrode is not accessible for easy cleaning. The first drawback can be eliminated, if samples are accurately plane-parallel, by moving the bellows to the outside (Fig. 7) or by substituting sliding contact fingers (Fig. 8). At high temperatures, bonding of electrodes to the sample occurs often, and expendable foil electrodes seem essential (Figs. 9 and 10).

Typical sample-holder sizes for standing-wave measurements in circular waveguide (TE_{11} mode) are shown in Fig. 11. The main features are a center-cooled junction to the standing-wave indicator, a thin-walled neck for thermal isolation, and the heated line section containing the sample. Holders for use to 600°C were made of fine silver with a silver-soldered shorting disk. Solid silver serves to 800°C. Platinum-clad (0.010-inch) steel has been used to 1000°C at 8500 Mc, and a holder bored from Pt rod has been used to 1200°C. Since there is always the possibility of samples sticking in these expensive holders because of high-temperature bonding, we have recently favored the resonance methods with foil cavities for temperatures above 500°C.

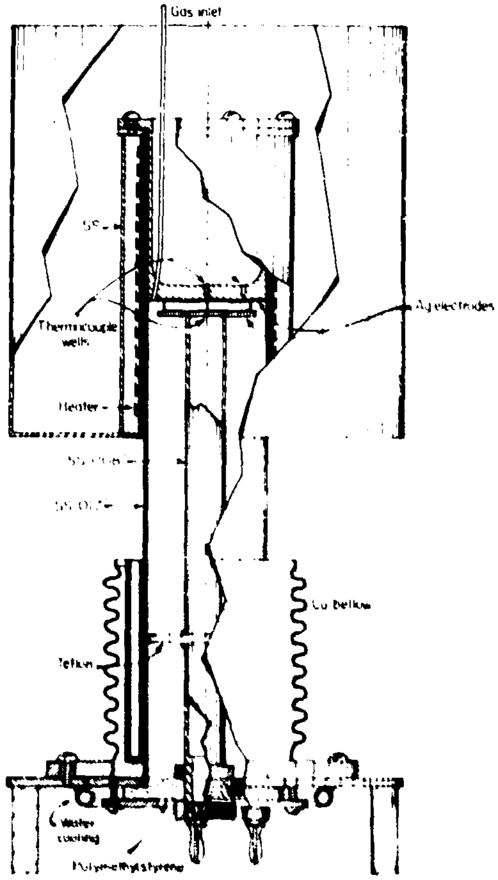


Fig. 7. Sample holder for 1-inch disks to 600°C.

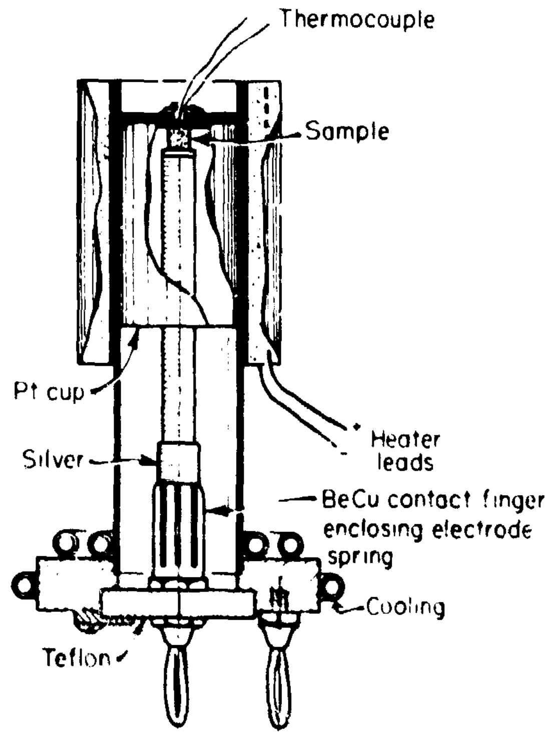


Fig. 8. Sample holder for small samples to 600°C.

Microwave-Resonance Method

All samples were first measured at 8520 Mc in Central Research Laboratories' Microwave Dielectrometer which used the standing-wave method[1] with traveling detector. Most of the high-temperature data were obtained using a resonance method with dielectric-filled cavities. For temperature runs in which the dielectric constant κ' changes appreciably, the standing-wave method is not ideal, because its sensitivity and accuracy vary with the electrical length of the sample. For maximum sensitivity of

1) "Dielectric Materials and Applications," A. von Hippel, Ed., The Technology Press of M.I.T. and John Wiley and Sons, New York, 1954, p. 63.

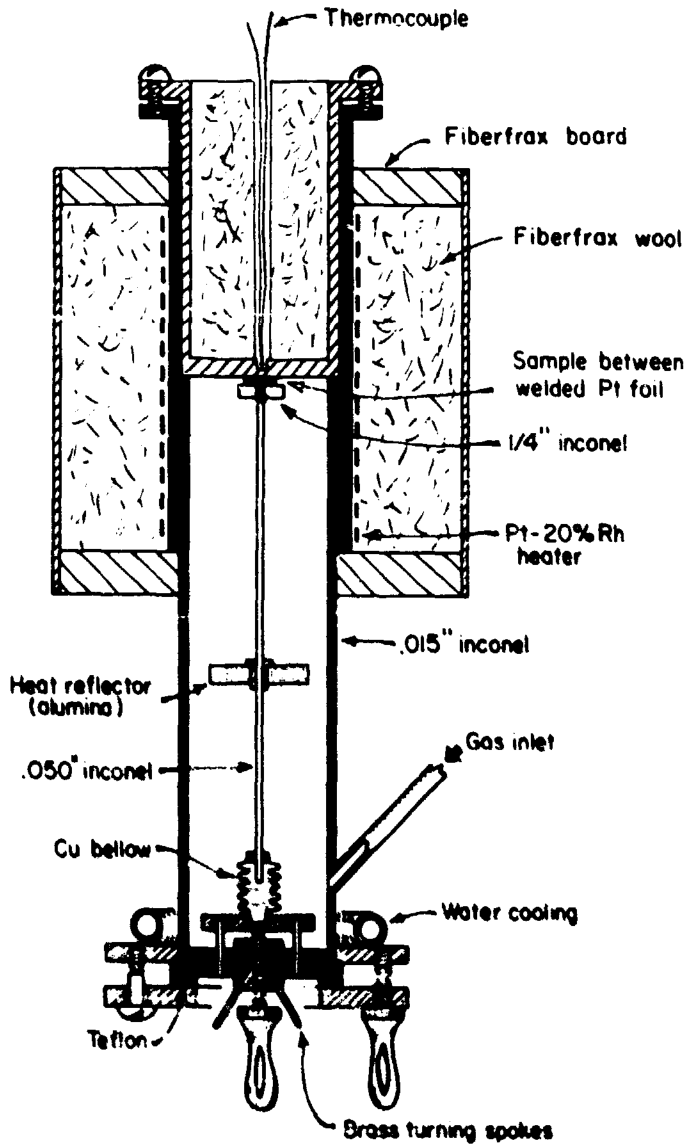
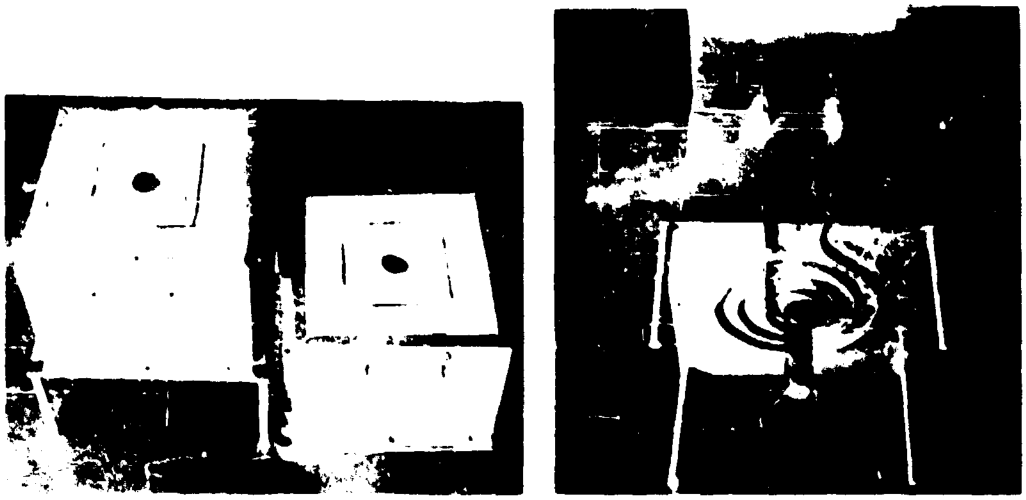


Fig. 9.

Sample holder for 5/8-inch disks to 1000°C.

measurement the sample length should not be a multiple of a half wavelength if measured against the shorted end of the line. A sample having an electrical length of $5\lambda/4$ at room temperature can change to a $3\lambda/2$ sample during the temperature run; the node shifts due to waveguide thermal expansion become important and limit the accuracy of measurements. Also conditions favorable for excitation of higher-order modes further limit the measurements. For



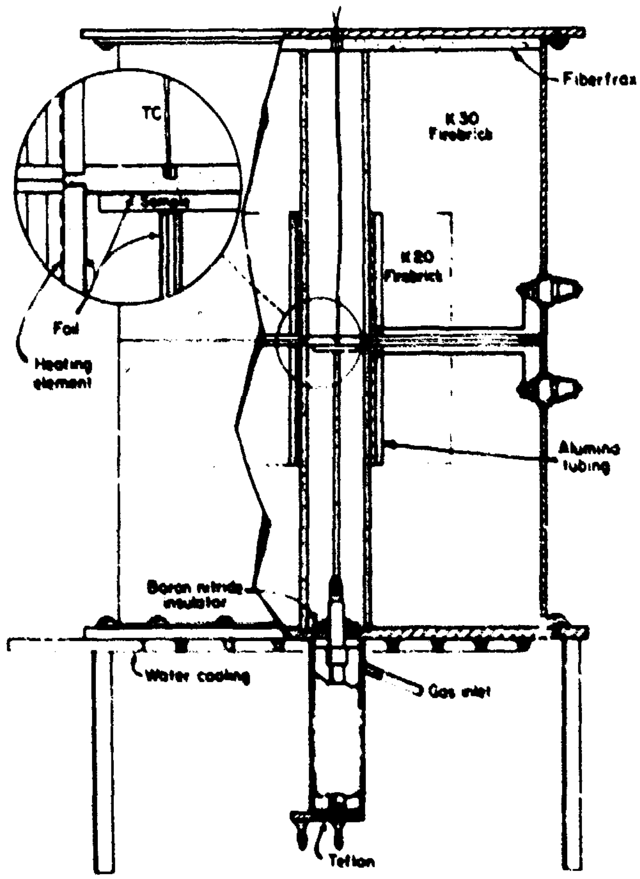


Fig. 10. Sample holder for 1-inch disks to 1400°C with Pt foil, or > 1400°C with Pt-Rh foil.

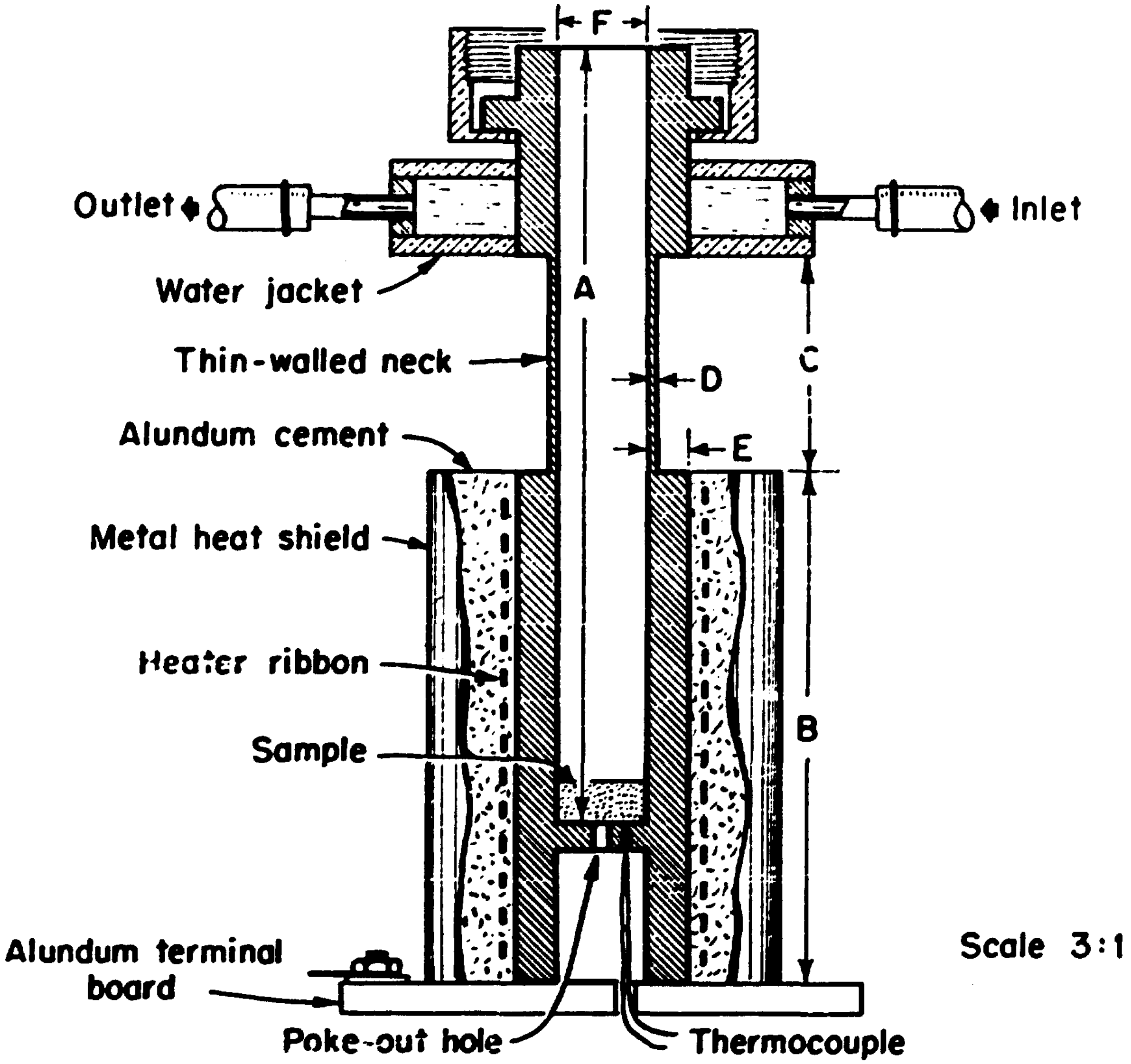


Fig. 11. Sample holder for 1-inch disks to 1400°C with Pt foil, or > 1400°C with Pt-Rh foil (longitudinal cross section).

	Nominal frequency	8.6×10^{9}	1.4×10^{10}	2.4×10^{10}	5×10^{10}
A.	Inside length	15.0 cm	7 cm	5.3 cm	3.1 cm
B.	Length heated	10.0 cm	4.2 cm	3.6 cm	2.4 cm
C.	Length of neck	3.6 cm	2.1 cm	1.0 cm	0.8 cm
D.	Neck thickness	0.030 in.	0.012 in.	0.010 in.	0.008 in.
E.	Wall thickness	1/8 in.	1/8 in.	1/16 in.	1/16 in.
F.	Inside diameter	1.0 in.	5/8 in.	3/8 in.	11/64 in.

the highest temperatures the standing-wave method can be expensive because of the Pt-line holders and the possibility of samples sticking in these holders.

For the resonant-cavity method samples were formed into dielectric-filled resonators by applying the following coatings:

to 850°C, silver paint, Dupont 4666 for precision κ' data;

to 850°C, silver foil, 0.001-inch wrap, 0.005-inch ends for lower loss;

to 1200°C, platinum paint, Hanovia 6926 for precision κ' data;

to > 1400°C, platinum foil 0.0015-inch thick.

The silver paint, applied in two coats, has an apparent resistivity relative to copper of 2. The platinum paint, even with three or four coats, has a resistance 20 times that of copper. For Ag and Pt foils no deviations from the expected resistivity factor of 0.95 and 6.16, respectively, were observed as long as the foil was free of wrinkles.

All samples were right cylinders, 0.999 ± 0.001 inch in diameter, with thickness between 5/8 and 7/8 inch and faces plane-parallel to 0.0005 inch. The foils were cut into disks, about 0,990-inch diam. with a punched hole 0.120-inch diam. in the center, and strips 8.35 mm long with widths equal to sample thickness plus 2 to 3 mm. Each strip was rolled onto the sample periphery and the ends joined by a lock seam such as used by sheet-metal workers. Disks were next placed against each face of the sample and the protruding edges of the strips folded inward to enclose the rim of the disks. Next the sample was hand pushed into a snug graphite tube fitted with graphite disks at each end. This graphite die was then heated to approximately 750°C for platinum and 500°C for silver and hot-pressed with a total force of about 1500 lbs, welding the joints between the three foil pieces and providing flat, unwrinkled metal faces. Since the aluminas and beryllias have higher thermal expansion than the carbon, the radial pressure may also

have welded the lock seam, but no effort was made to check this effect.

For the temperature run, a graphite cup was placed over each end of the metal-clad sample. An axial hole in each cup allowed a coupling loop to be inserted near the iris in the platinum foil, while a second hole in one cup provided space for a Pt-Rh thermocouple. The graphite served to isolate the loops electrically. The entire assembly fitted into $1\frac{1}{4}$-inch i.d., $1\frac{1}{2}$-inch o.d. high-alumina tube. The heating element was a Pt-20% Rh ribbon wound in the center $3\frac{1}{2}$-inch portion of the 12-inch tube. At 1400°C the power input was 550 watts. Each end of the oven was closed with Fiberfrax, and a light flow of prepurified nitrogen was maintained. Oven insulation was a combination of fire brick and Fiberfrax wool.

Theory. For a right cylinder with diameter greater than height the lowest frequency of resonance is for the TM_{010} mode,[2)]

$$\lambda = 1.30637\, D\sqrt{\kappa'}\,, \tag{1}$$

and the Q of the cavity with lossless dielectric is

$$Q_w = \frac{10^4\sqrt{\lambda/S}}{1 + \frac{0.384\lambda}{h}}\,, \tag{2}$$

where λ is the free-space wavelength in cm corresponding to the resonant frequency, D the diameter and h the height in cm, and S the resistivity of the metal walls relative to copper. The total losses are measured by the width, $\Delta\lambda$, of the resonance curve between half-maximum power points. The dielectric loss tangent is the difference between the apparent loss tangent of the cavity and $1/Q_w$

$$\tan\delta = \frac{\Delta\lambda}{\lambda} - \frac{1}{Q_w}\,. \tag{3}$$

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2) Adapted from "Reference Data for Radio Engineers," 4th ed., ITT Publication, 1962.

The advantage of low operation frequency for the TM_{010} mode is offset by the fact that warping of the face foil can degrade the Q appreciably. The E field is a maximum along the axis and shows fringing effects at the iris; the H field is zero at the axis, and it is thus difficult to couple into the cavity. Satisfactory operation can be achieved by offsetting the iris from the axis. A temperature run on one sample showed no appreciable difference in data between the TM_{010} and the TE_{111} mode at 40% higher frequency.

All samples were measured using the TE_{111} mode,[2)] the next lowest frequency mode for which

$$\lambda = \frac{\sqrt{\kappa'}}{\left(\frac{0.3434725}{D^2} + \frac{0.25}{h^2}\right)^{1/2}} , \qquad (4)$$

and

$$Q_w = \frac{1.31 \times 10^4 h}{\sqrt{\lambda S}} \left(\frac{2.39\, h^2 + 1.730\, D^2}{3.39 \frac{h^3}{D} + 0.73\, Dh + 1.73\, D^2} \right). \qquad (5)$$

Table 3 lists the values of $1/Q_w = \tan \delta_w$ for h values ranging from 1.5 to 2.0 cm and wavelengths from 6 to 10 cm for a diameter of 2.540 cm, metal-foil walls at room temperature. For elevated temperatures the resistivity change of the metal enters, and the tabulated values must be multiplied by $\sqrt{1 + (dK/dT)\Delta T}$ based on a linear change in resistivity with temperature (Fig. 12).

For low-loss samples, the loss measured as a resonant cavity at room temperature agreed with the 8.5-Gc value within limits of error. Some of the higher-loss materials showed appreciably lower loss in the 4-kMc region of resonance measurements.

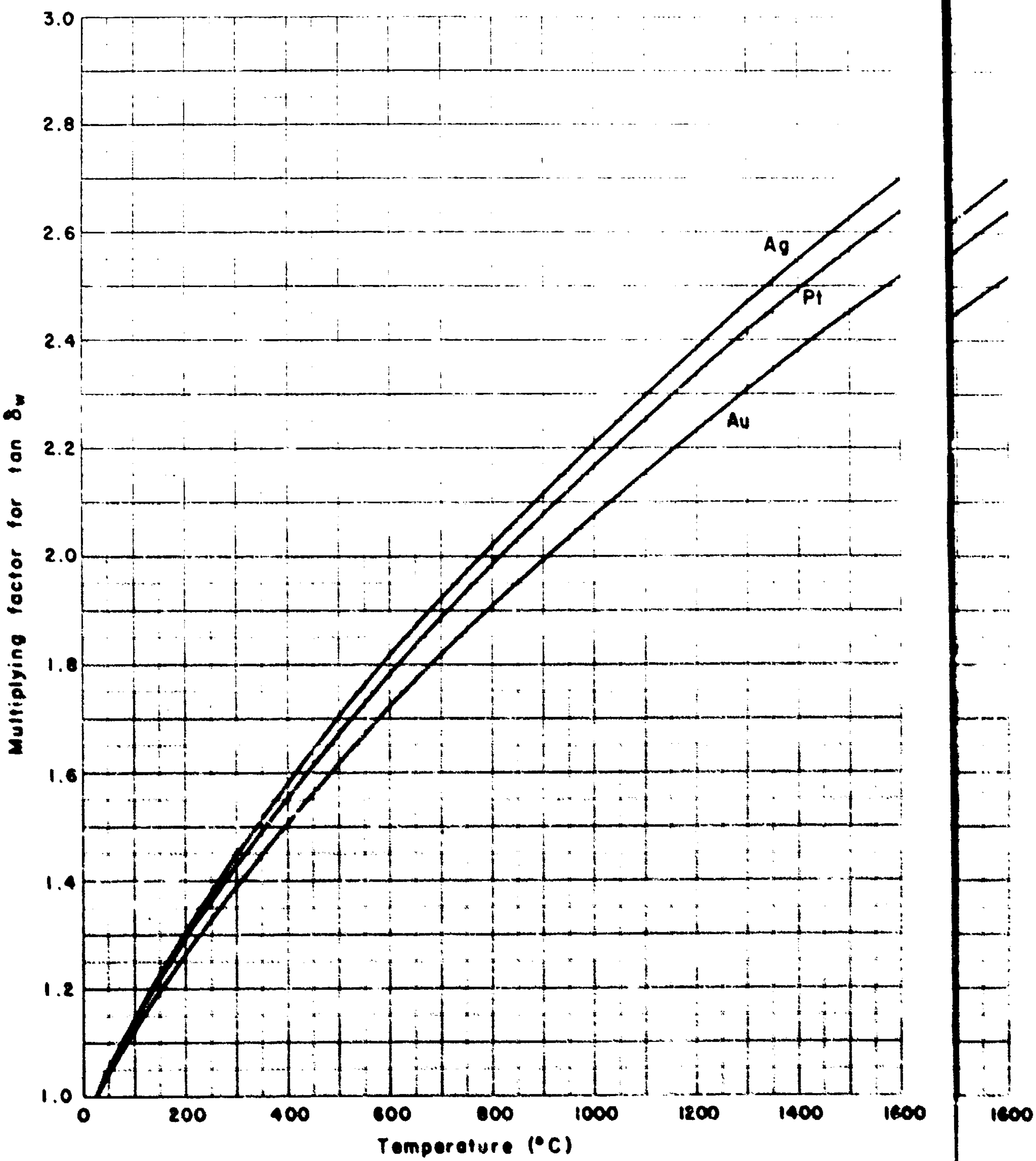


Fig. 12. Multiplying factor for tan δ_w for various metals.

Table 3. Wall loss in TE_{111} metal-foil cavity.
(Diameter D = 2.54 cm)

Free-space wavelength (cm)	Height (cm)	$\tan\delta \times 10^{4}$		
		Platinum	Silver	Gold
10	1.5	4.46		2.14
	1.6	4.25		2.04
	1.7	4.08		1.96
	1.8	3.93		1.88
	1.9	3.80		1.82
9	1.5	4.23		2.03
	1.6	4.03		1.93
	1.7	3.87		1.855
	1.8	3.73	1.463	1.79
	1.9	3.61	1.418	1.73
8	1.5	3.99	1.567	1.91
	1.6	3.80	1.491	1.82
	1.7	3.64	1.43	1.745
	1.8	3.51	1.377	1.69
	1.9	3.40	1.334	1.63
7	1.5	3.73	1.463	1.79
	1.6	3.56	1.40	1.71
	1.7	3.41	1.34	1.635
	1.8	3.29	1.29	1.57
	1.9	3.18	1.25	1.525
6	1.5	3.45	1.35	1.653
	1.6	3.29	1.29	1.577
	1.7	3.16	1.24	1.575
	1.8	3.04	1.19	1.458
	1.9	2.95	1.16	1.414

Table 4. c in $\kappa'_{corr.} = \kappa'_{meas.} - c\kappa'_{meas.}$

T^oC	Al_2O_3	BeO	SiO_2
200	3.1×10^{-3}	2.39×10^{-3}	2.0×10^{-4}
400	5.6	5.75	4.25
600	9.0	9.15	6.50
800	12.5	13.28	8.72
1000	16.5	17.48	11.2
1200	20.1	21.7*	13.4*
1400	24.1	25.85*	-
1600	28.0	-	-

* Extrapolated.

The measured value of κ' determined from Eq. 1 or 4, considering D and h to be temperature-invariant, gives the effective κ' for a dielectric-filled cavity with thin walls. The tables of data included in this report list these effective values unless otherwise noted. The correct material parameters are obtained, when the thermal expansion is known, by using the corrected values of D and h in Eqs. 1 and 4. For isotropic materials the corrected value of κ' is the tabulated value divided by $1 + 2\alpha\Delta T$, where α is the linear expansion coefficient, higher-order terms being neglected. The reciprocal of $1 + 2\alpha\Delta T$ can be written as $1 - c$. Table 4 lists c for alumina and beryllias based on expansion data given by Ryshkewitch[3] and other sources.

For Coors RR and AD-995 alumina, Brush beryllia, and American Optical quartz, the corrected values of κ' are listed in the figures.

3) E. Ryshkewitch, "Oxide Ceramics," Academic Press, New York and London, 1960.

Equipment. The signal generators were manually tuned klystrons having band widths that limited the accuracy of measurements as samples became lossy with temperature. A backward wave oscillator with leveler (Paradynamics 851A) was recently purchased and used for rapid searching for resonance. In its present form the modulated output contains frequency modulations that are excessive for low loss measurements. The CRL Dielectrometer was used as a coaxial wavemeter at frequencies < 5.5 Gc. For higher frequencies, a 5/8-inch diam. coaxial standing-wave detector achieved about the same resolution, 1 part in 30,000. A block diagram of the equipment is given in Fig. 13.

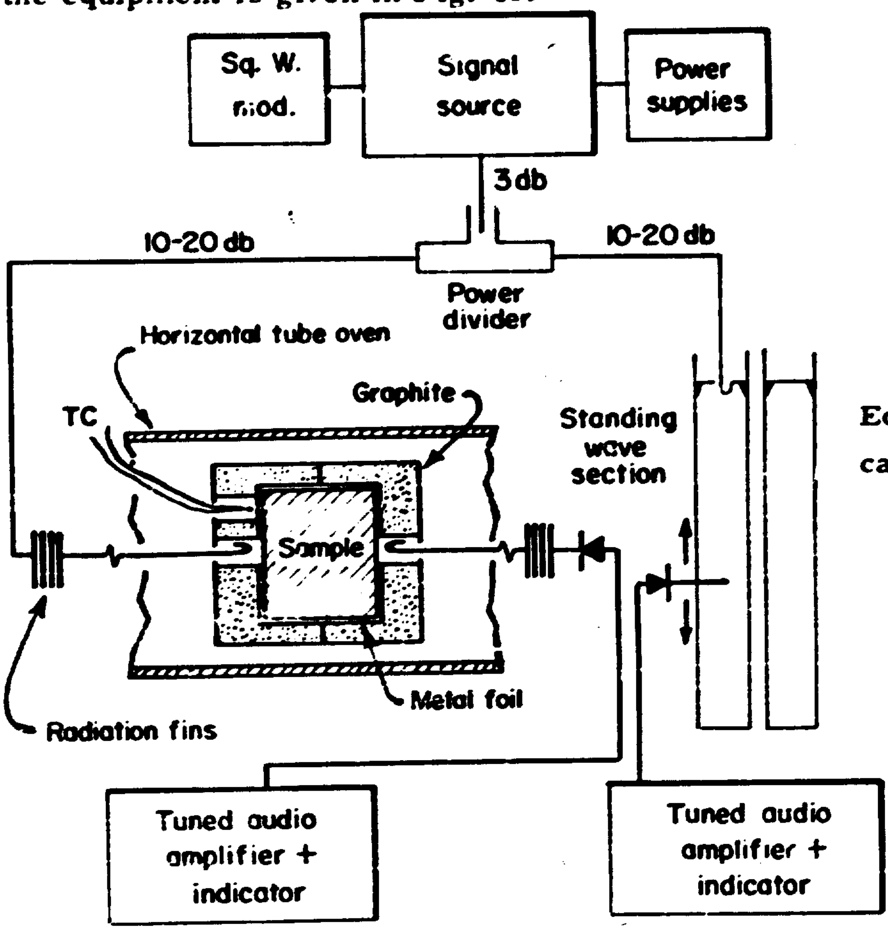


Fig. 13.

Equipment for resonant-cavity measurements.

Four-Terminal Equipment

Depending on the charge carriers and their mobilities, two-terminal samples may exhibit electrode polarization. This effect can be better studied

with four-terminal samples measured in the potentiometer method of Fig. 11. With all indicators balanced to zero, the impedance of each sample section is related to its corresponding balanced impedance by the ratio R_1/R_2. A block diagram of equipment for measuring sample impedance up to about 10 meg-ohms at 100 cycles (10^4 ohms at 100 kc) is shown in Fig. 15. The cathode-follower detector is shown schematically in Fig. 16. The physical layout is illustrated in Fig. 17, which shows the sample oven above the coaxial switch and balancing impedances. The series resistors plug into shield compartments in the switch assembly. Lack of suitable samples and the long balancing process have limited the usefulness of this type of measurement.

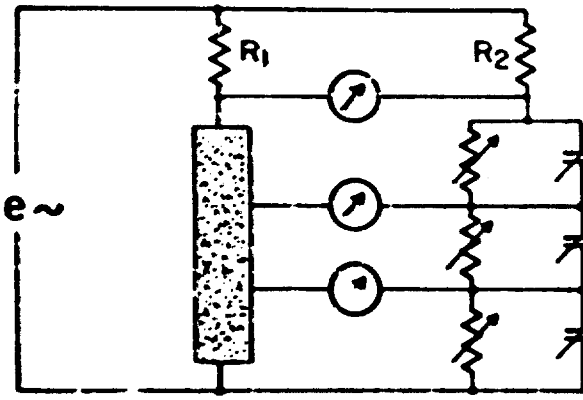


Fig. 14. Four-terminal measurements.

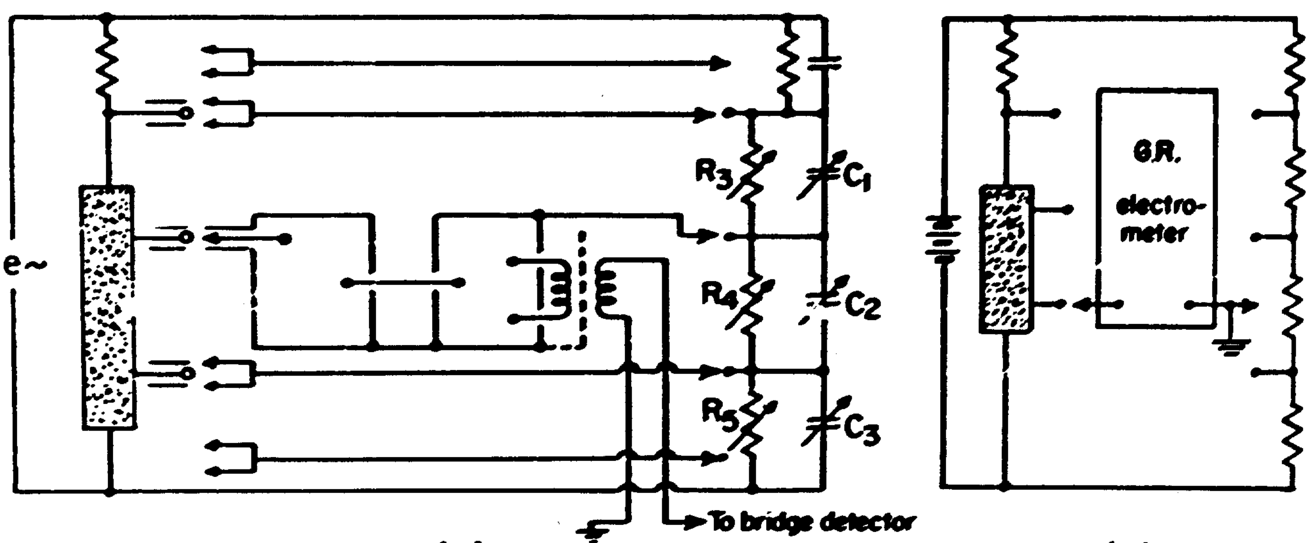


Fig. 15. Four-terminal equipment for (a) a.c. and (b) d.c. R_3, R_5 are 1-ohm steps to 150 K, wire-wound; R_4 is a 10-ohm step to 10 megohms, deposited carbon; C_1, C_2, C_3 are air capacitors to 0.001 pf, polystyrene, to 1 μf.

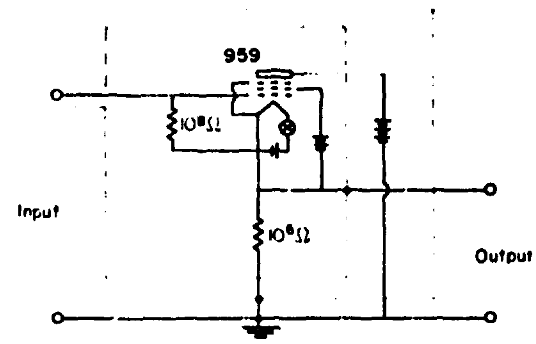


Fig. 16.

Schematic of cathode-follower detector. Impedance at input is 10^9 ohms, 0.05 pf.

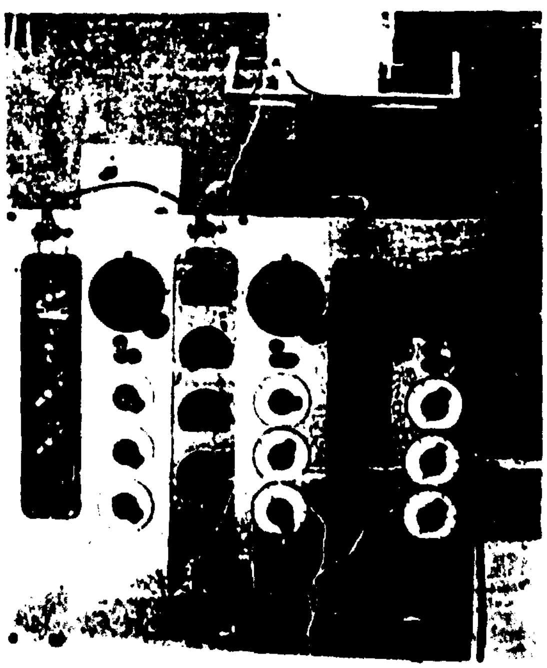


Fig. 17.

Physical layout of four-terminal equipment.

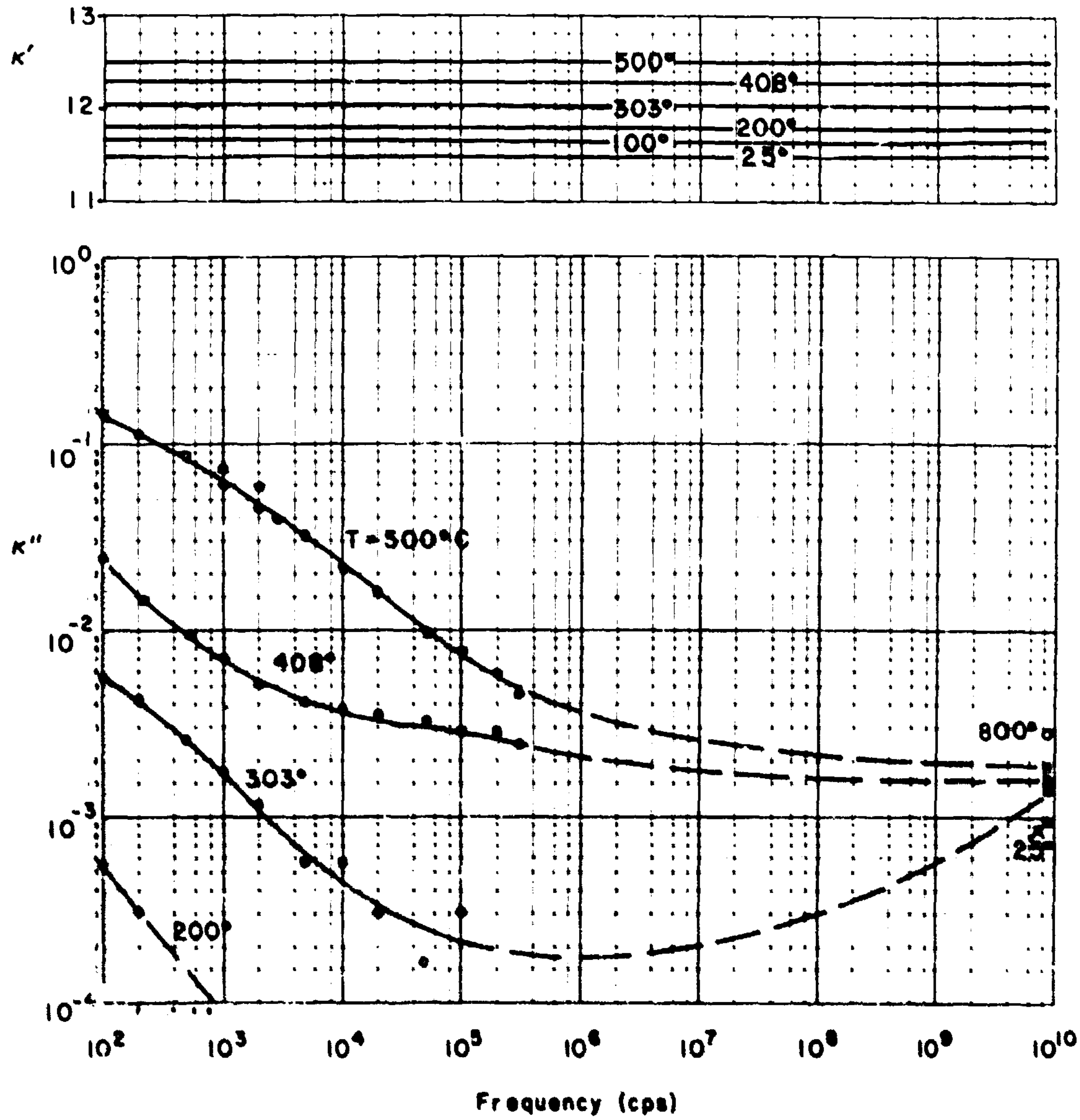
II. Frequency-Response Characteristics

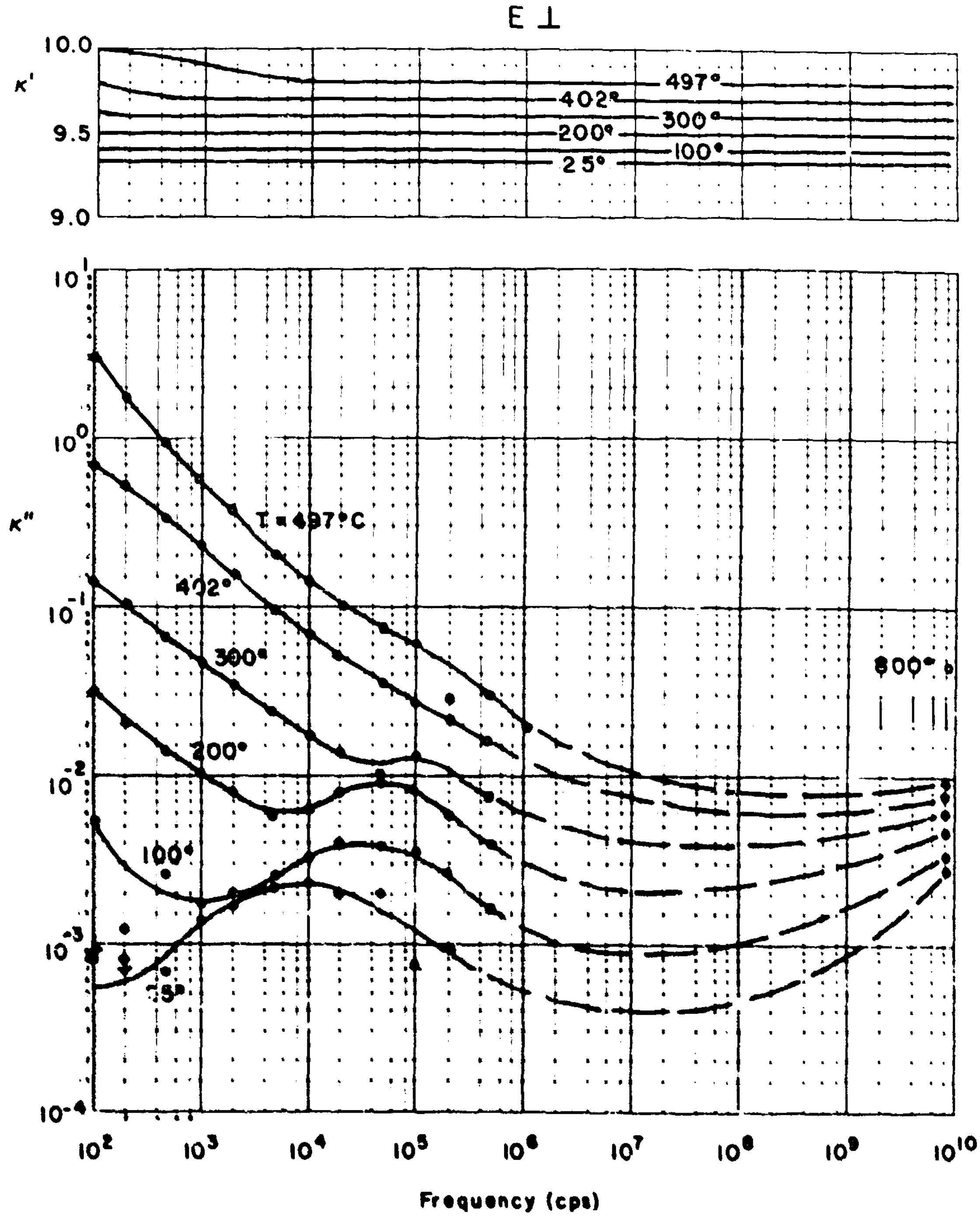
The following pages are graphs of dielectric constant, κ', loss factor, κ'', and volume conductivity, σ, versus frequency and temperature.

Al_2O_3 single-crystal sapphire, low-frequency peak dispersion due to silver diffusion. To be re-evaluated to higher temperatures with platinum electrodes.

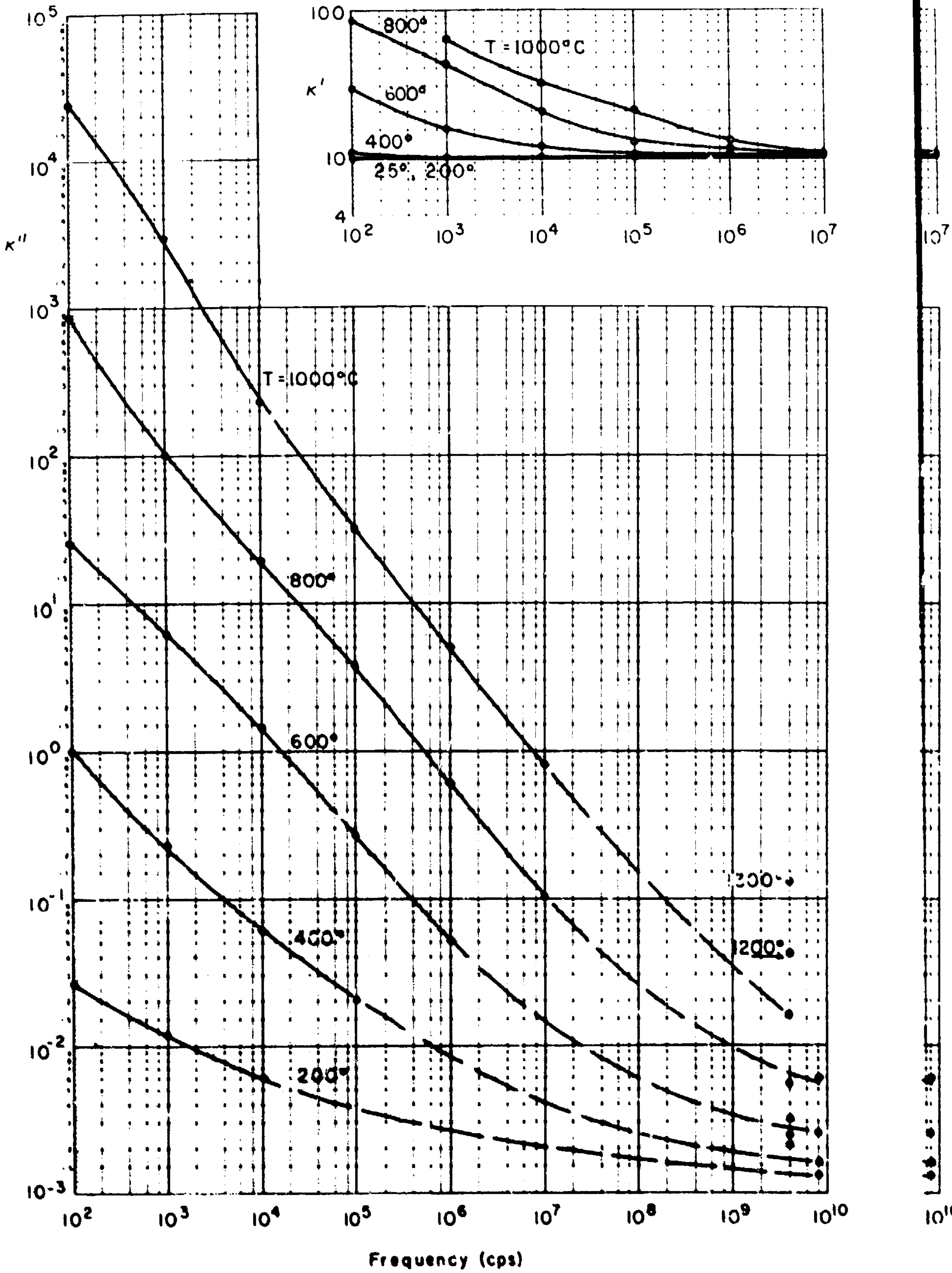
The Linde Air Products Co.

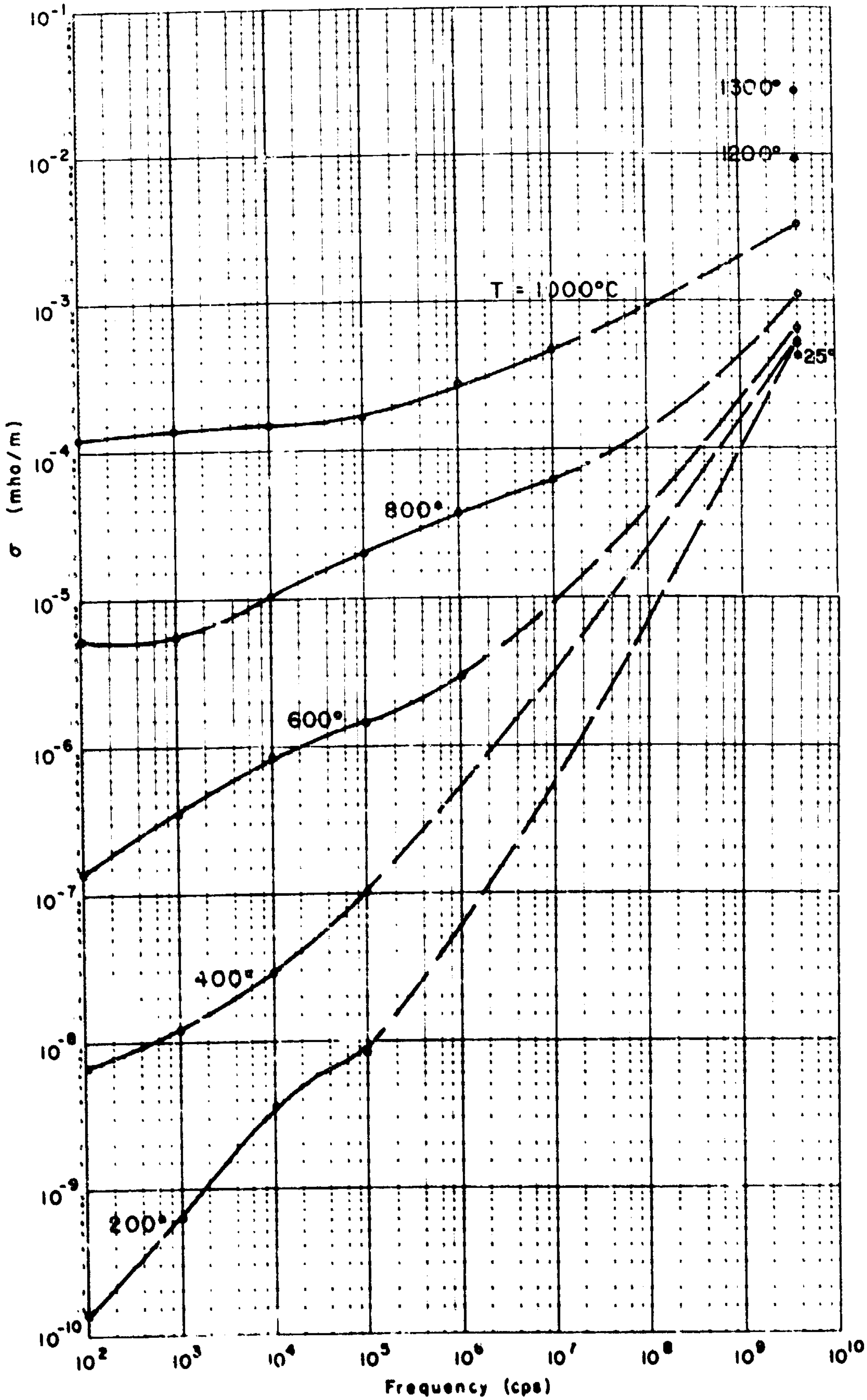
E 11




E ⊥
κ'
10.0
9.5
9.0
497°
402°
300°
200°
100°
25°
κ"
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
T = 497°C
402°
300°
200°
100°
25°
800°
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
Frequency (cps)

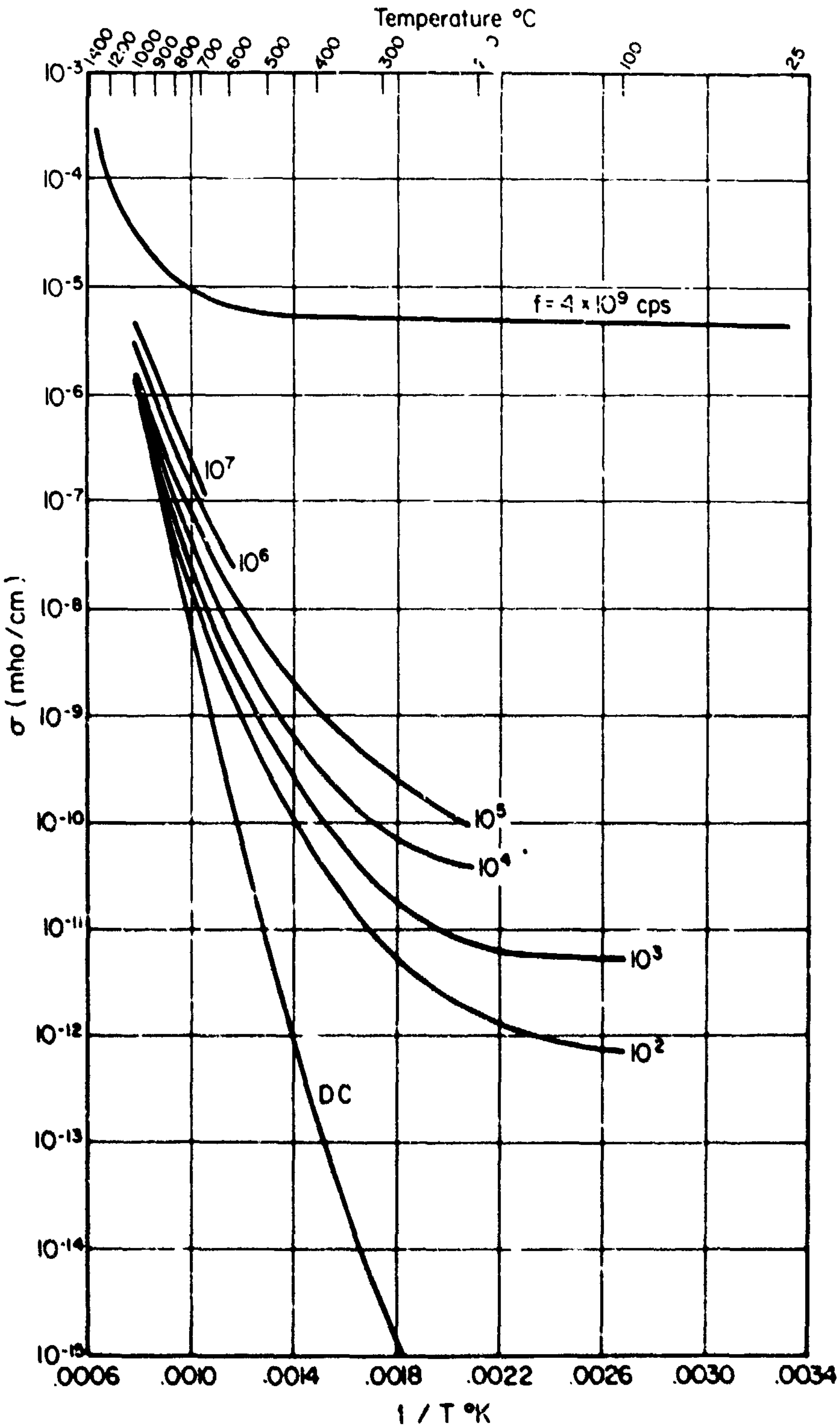
Al$_2$O$_3$ 99% ceramic, Coors Porcelain Co. AD-99



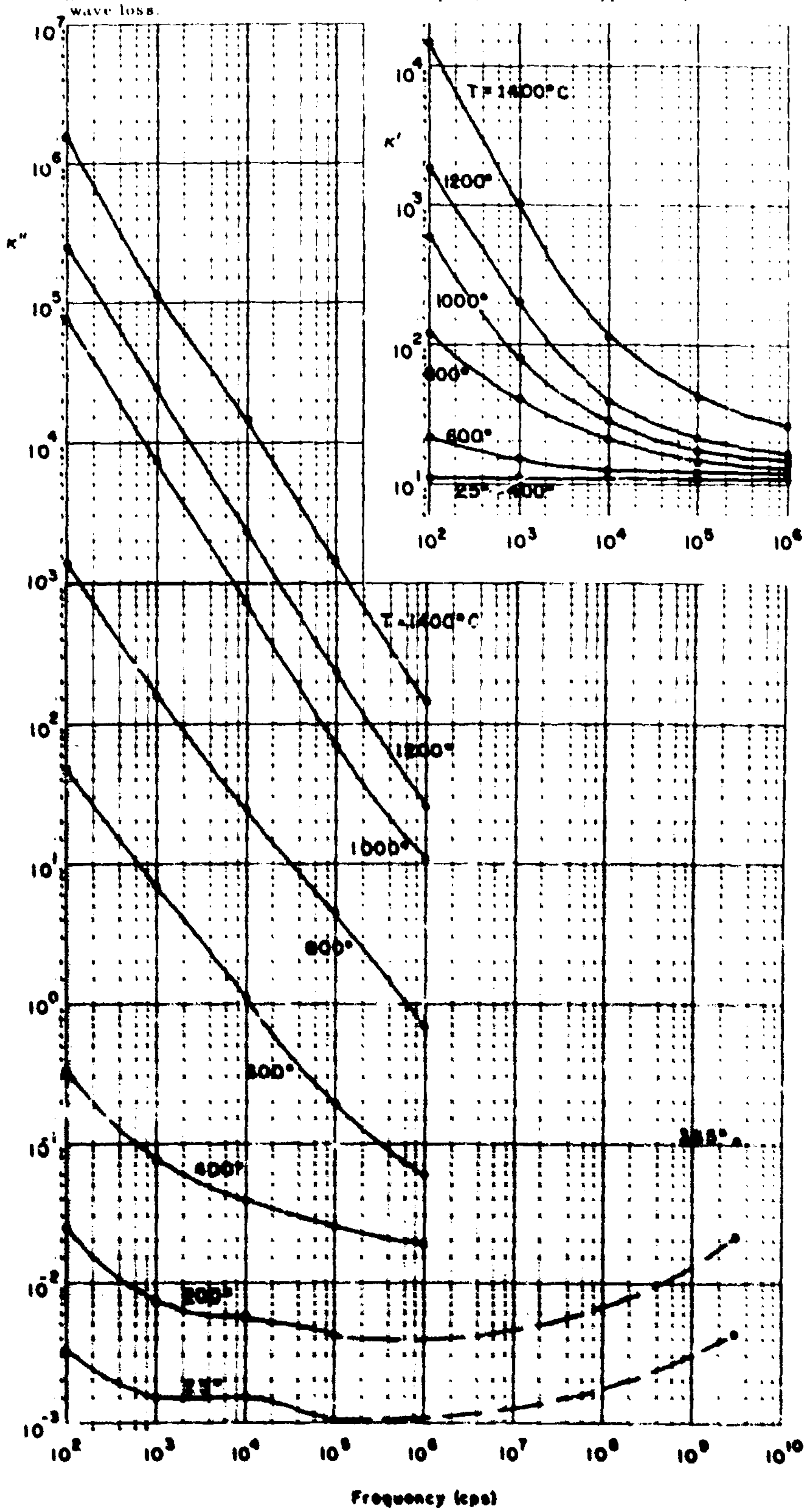


1300°
1200°
T = 1000°C
800°
600°
400°
200°
25°
σ (mho/m)
Frequency (cps)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}

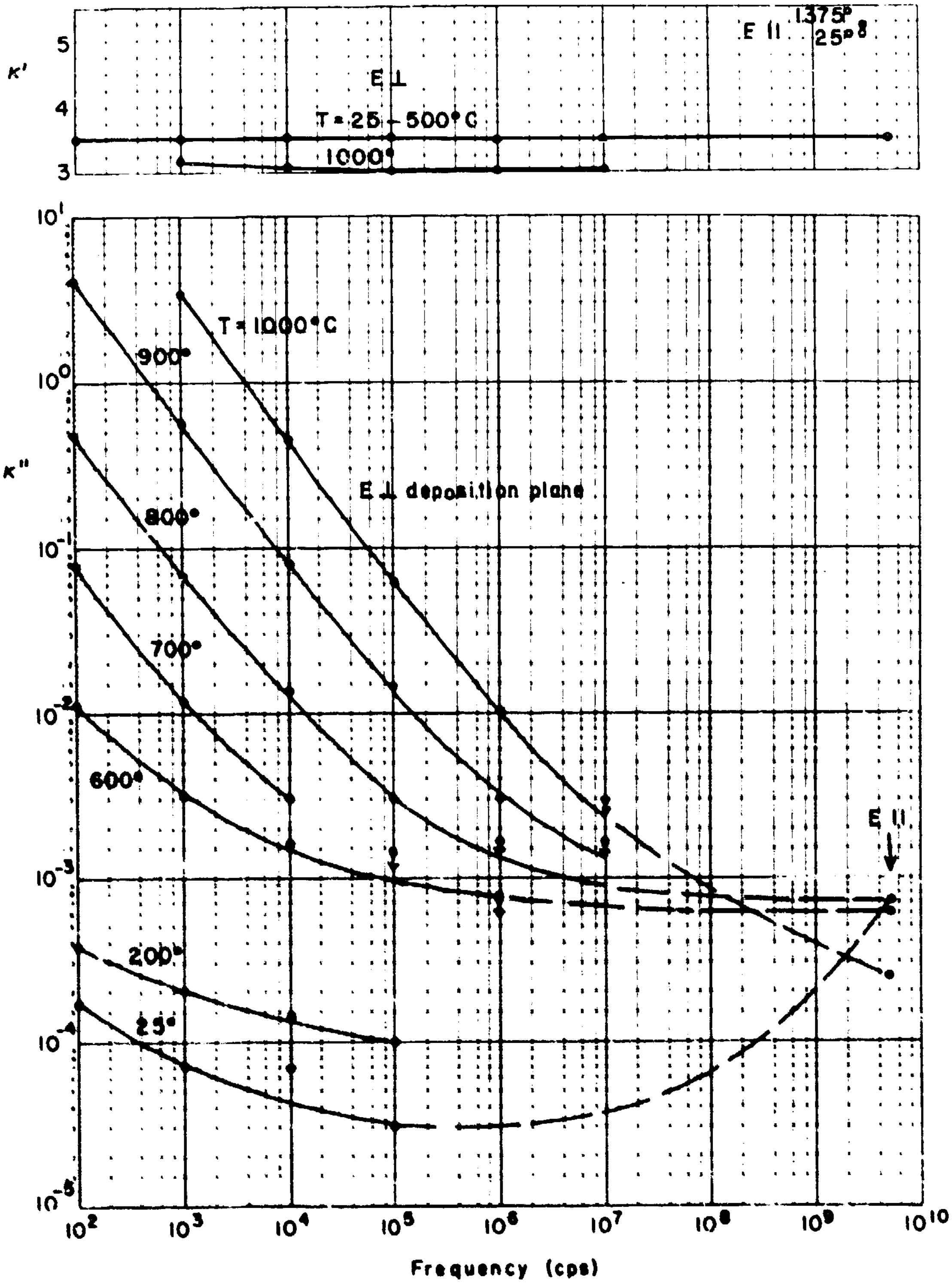
Al_2O_3, 99% ceramic, Coors Porcelain Co., AD-99.

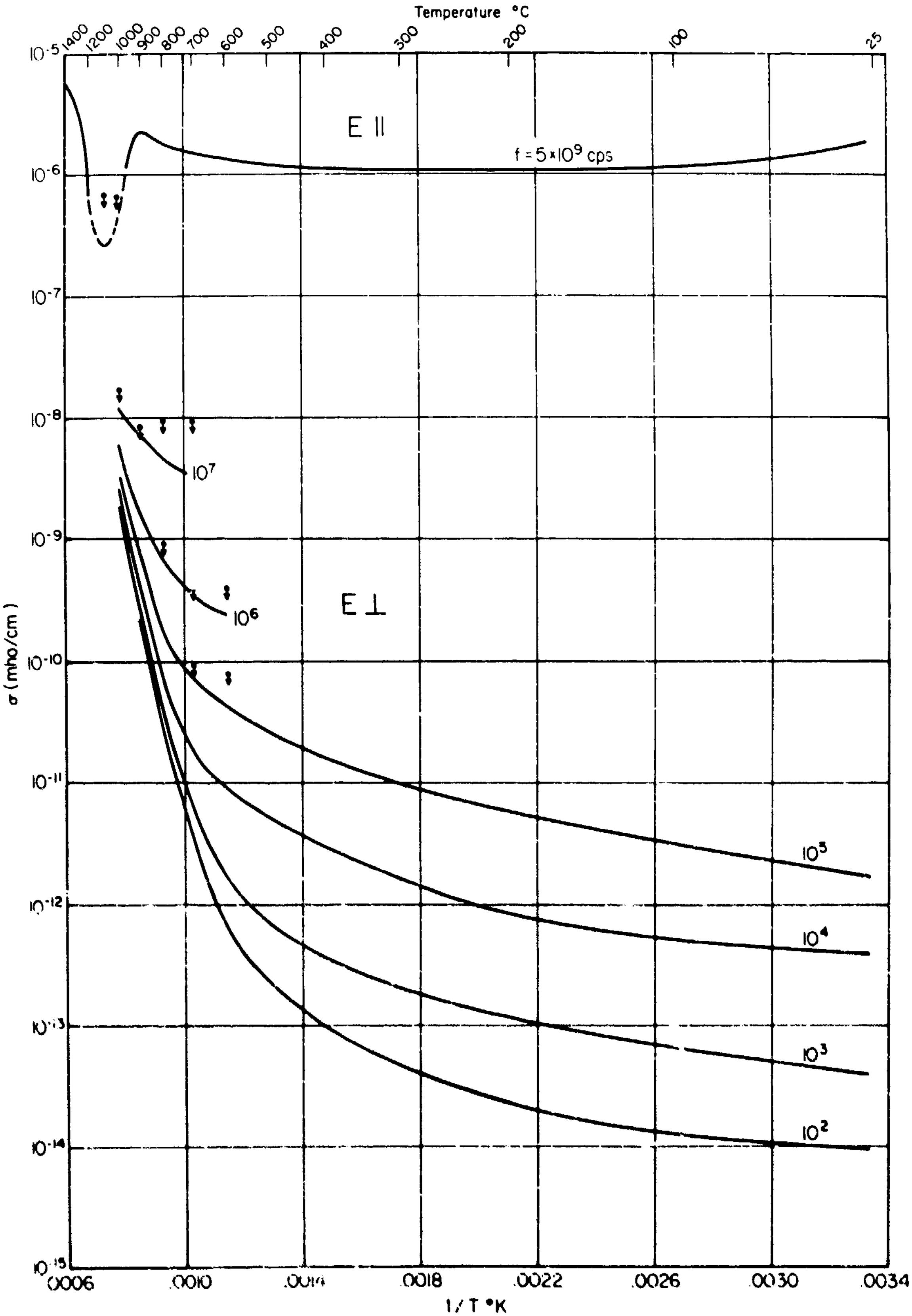


Al_2O_3 ceramic, General Electric Co. "Lucalox." Curves show data on samples bought in 1963. Tabulation of data on earlier samples, 1962, show appreciably lower micro-wave loss.



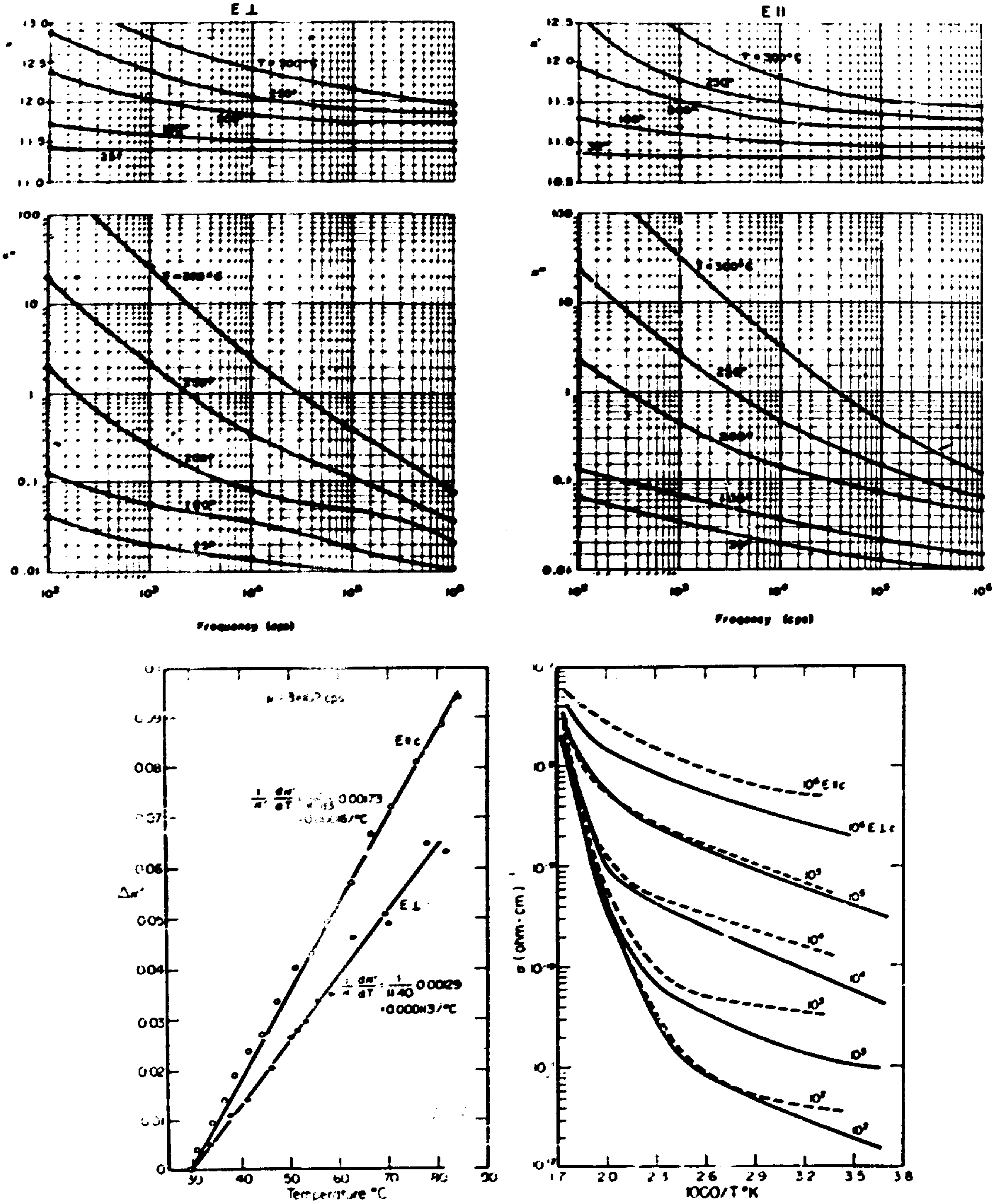
BN, pyrolytically deposited, High-Temperature Materials, Inc., "Boralloy." The microwave data show a small peak possibly due to loss of impurities (perhaps OH ions) at about 800°C. Graphite electrodes and prepurified N_2 used in low-frequency measurements which showed variations among different samples.



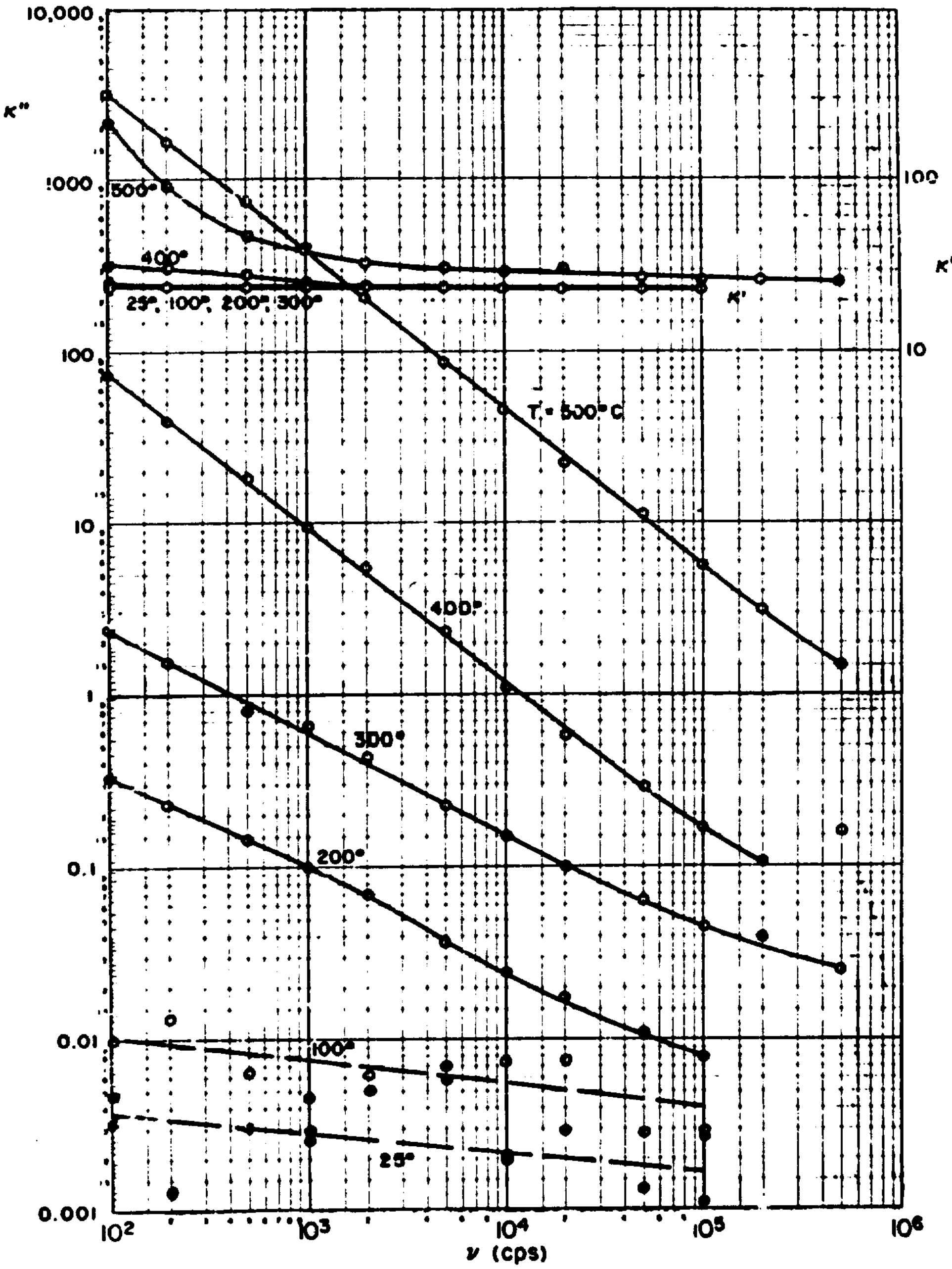


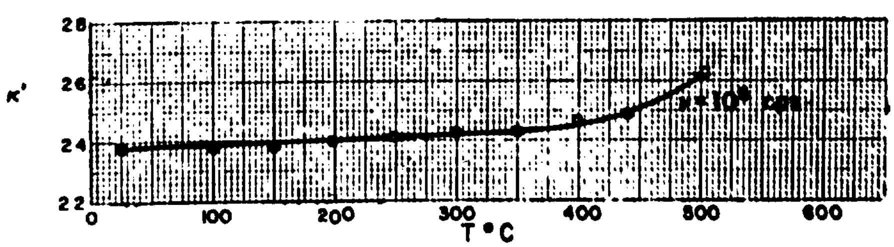
Temperature °C
1400
1200
1000
900
800
700
600
500
400
300
200
100
25
E ||
f = 5×10⁹ cps
10⁷
10⁶
E ⊥
10⁵
10⁴
10³
10²
σ (mho/cm)
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻¹²
10⁻¹³
10⁻¹⁴
10⁻¹⁵
.0006
.0010
.0014
.0018
.0022
.0026
.0030
.0034
1/T °K

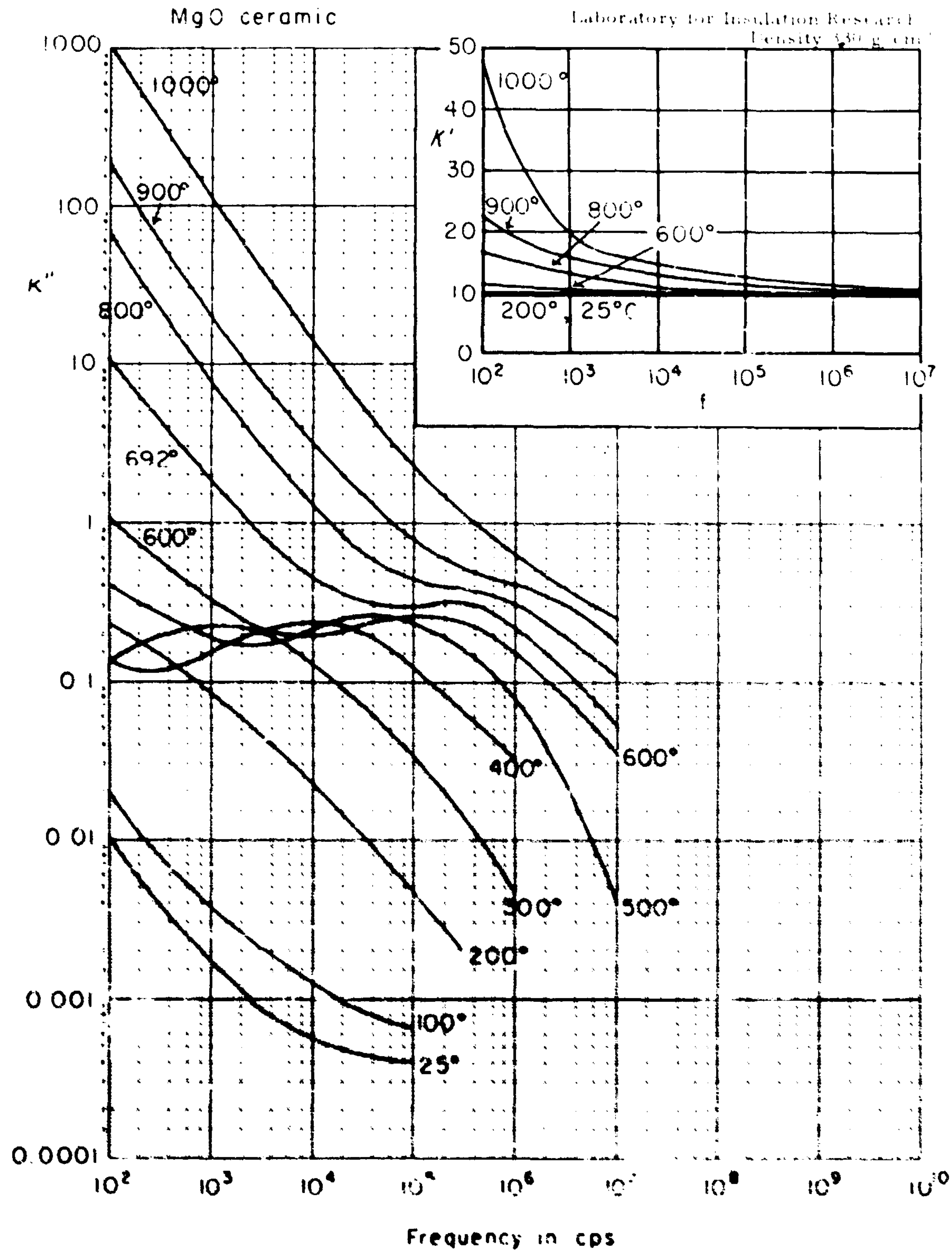
Cr_2O_3 **single crystal, The Linde Air Products Co.**

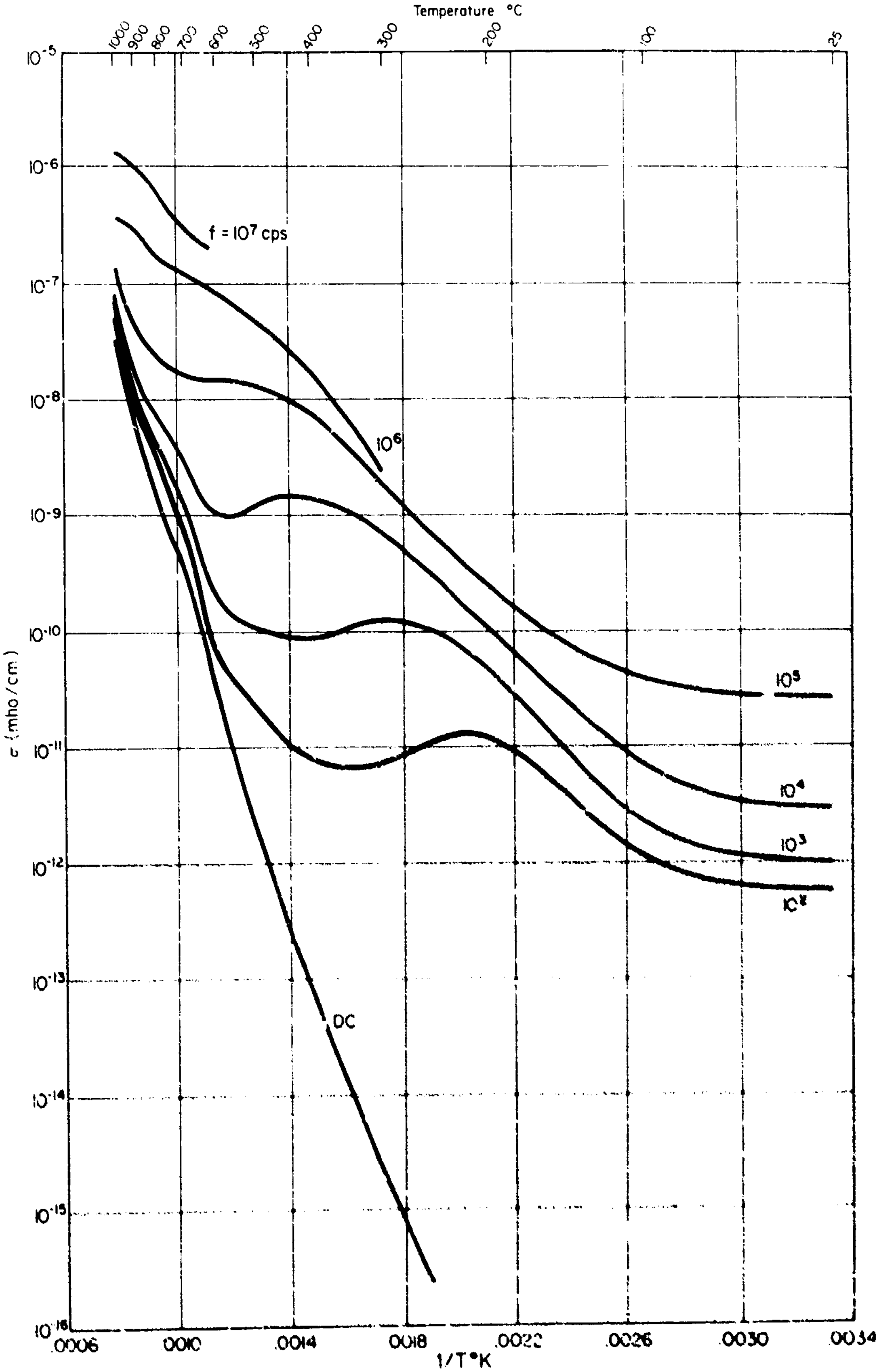


$LaAl_3$ single crystal Laboratory for Insulation Research



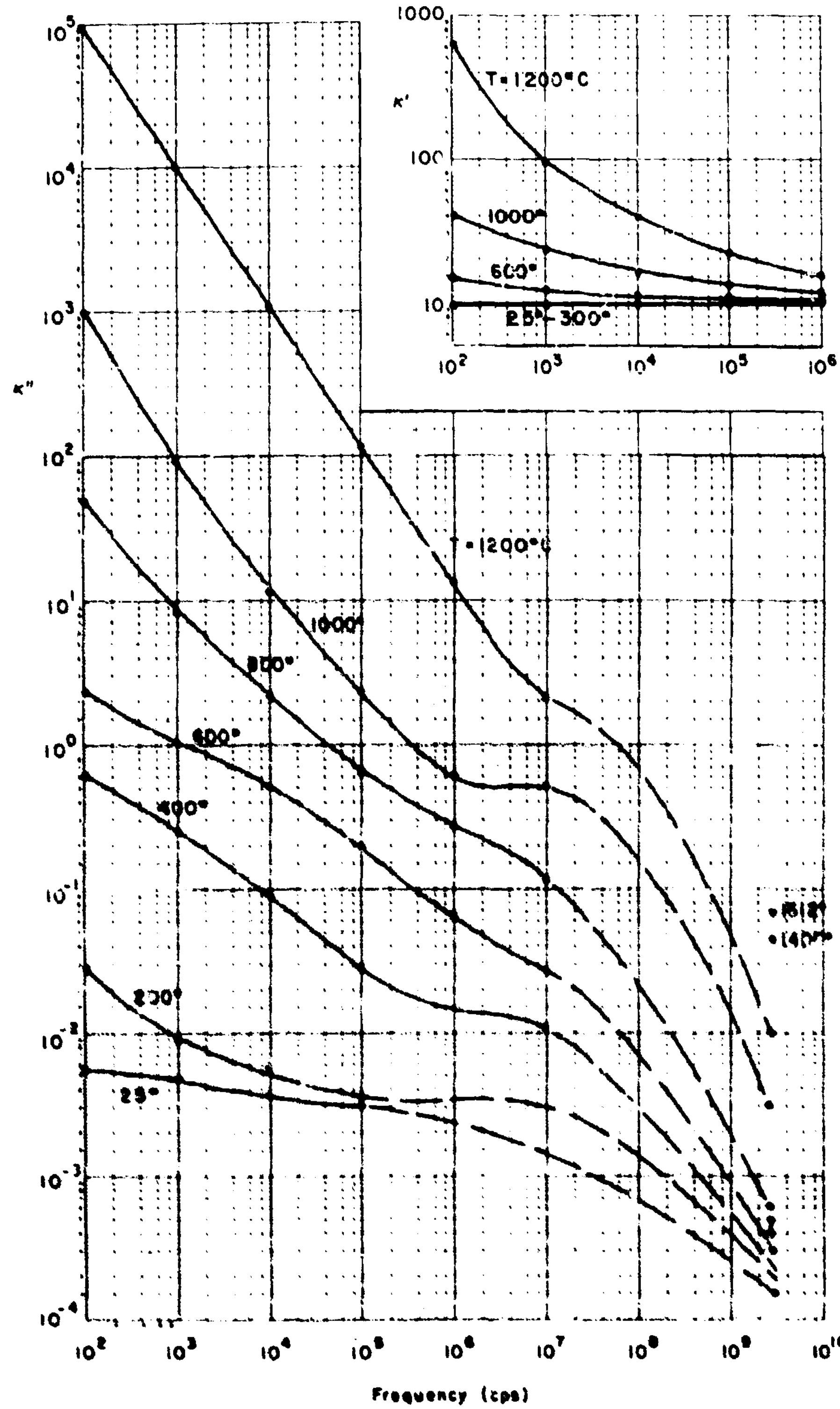


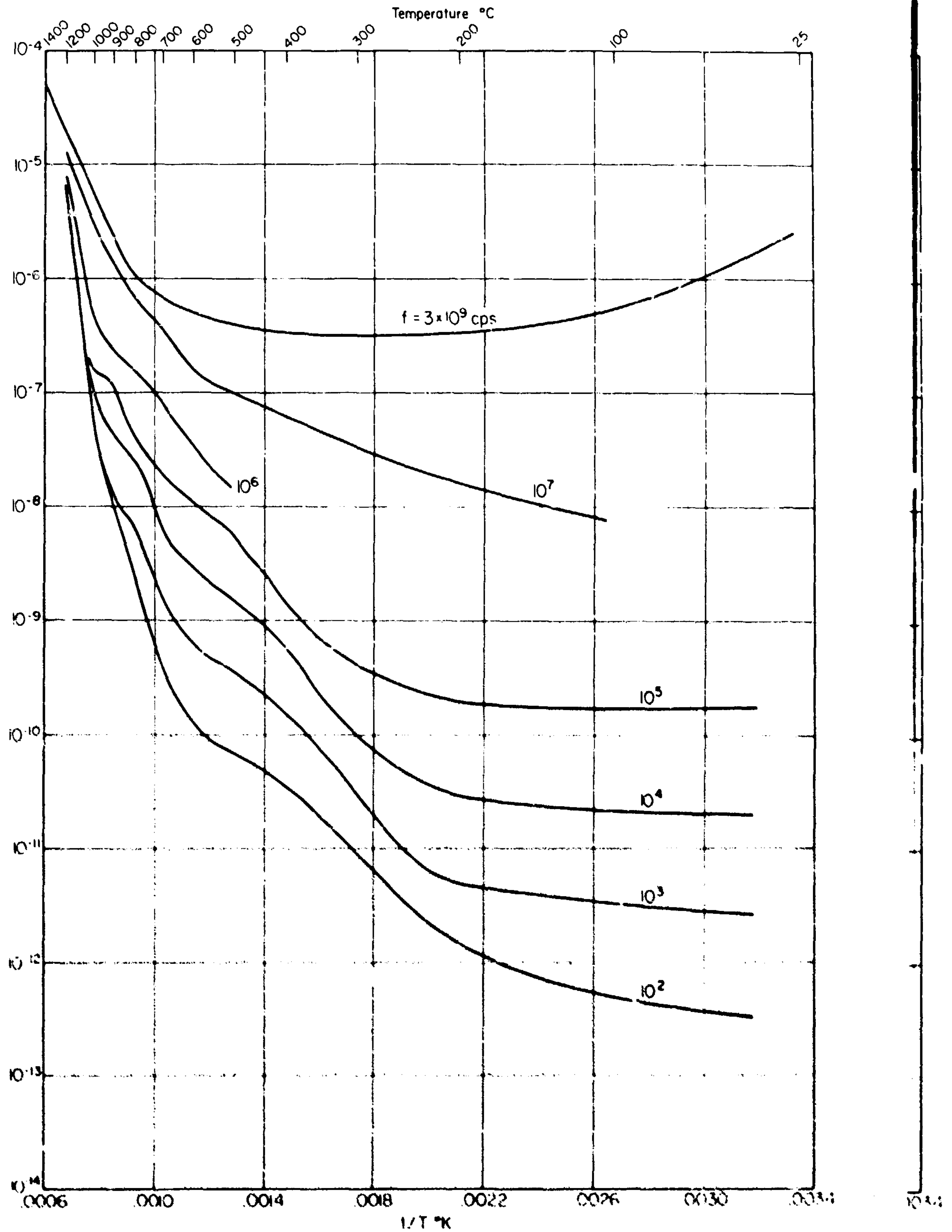

MgO ceramic
Laboratory for Insulation Research
1000
100
10
1
0.1
0.01
0.001
0.0001
κ''
1000°
900°
800°
692°
600°
400°
600°
300°
500°
200°
100°
25°
50
40
30
20
10
0
κ'
1000°
900°
800°
600°
200°, 25°C
f
Frequency in cps



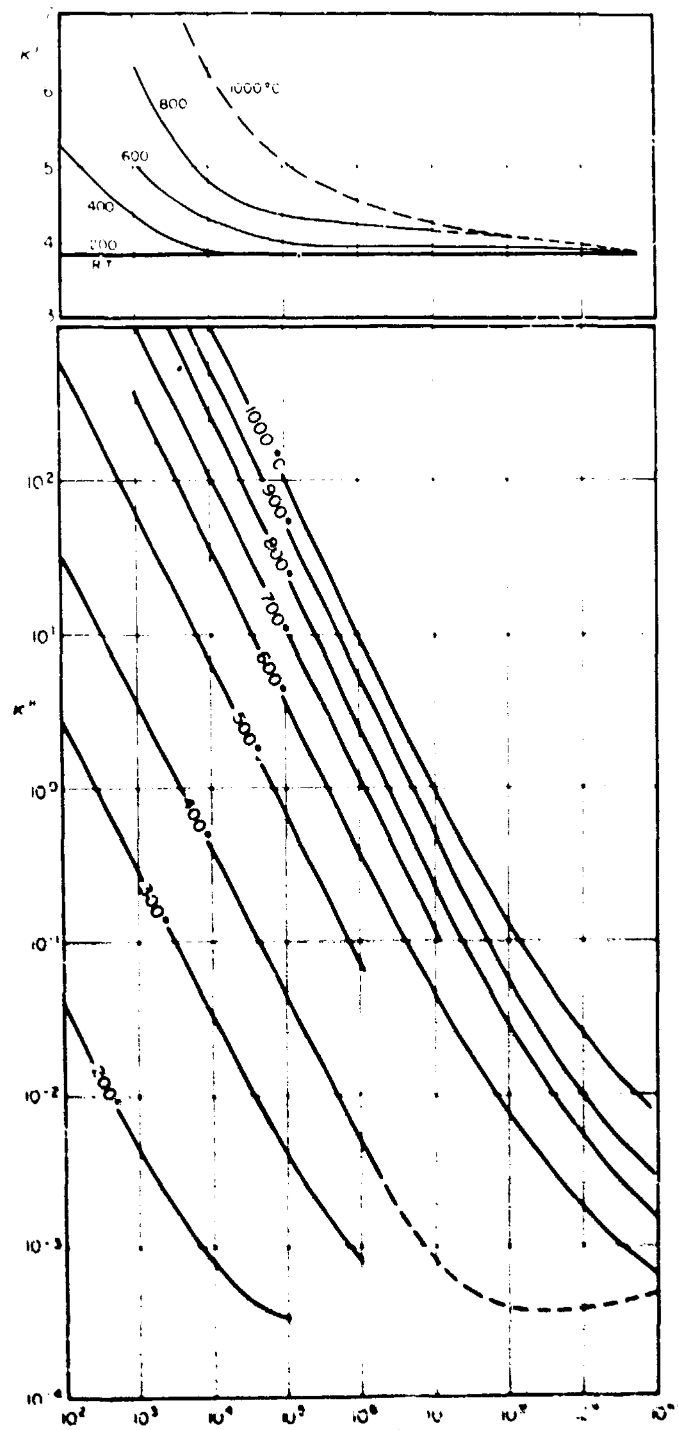
Temperature °C
1000
900
800
700
600
500
400
300
200
100
25
f = 10^7 cps
10^6
10^5
10^4
10^3
10^2
DC
σ (mho/cm)
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
10^-14
10^-15
10^-16
.0006
.0010
.0014
.0018
.0022
.0026
.0030
.0034
1/T °K

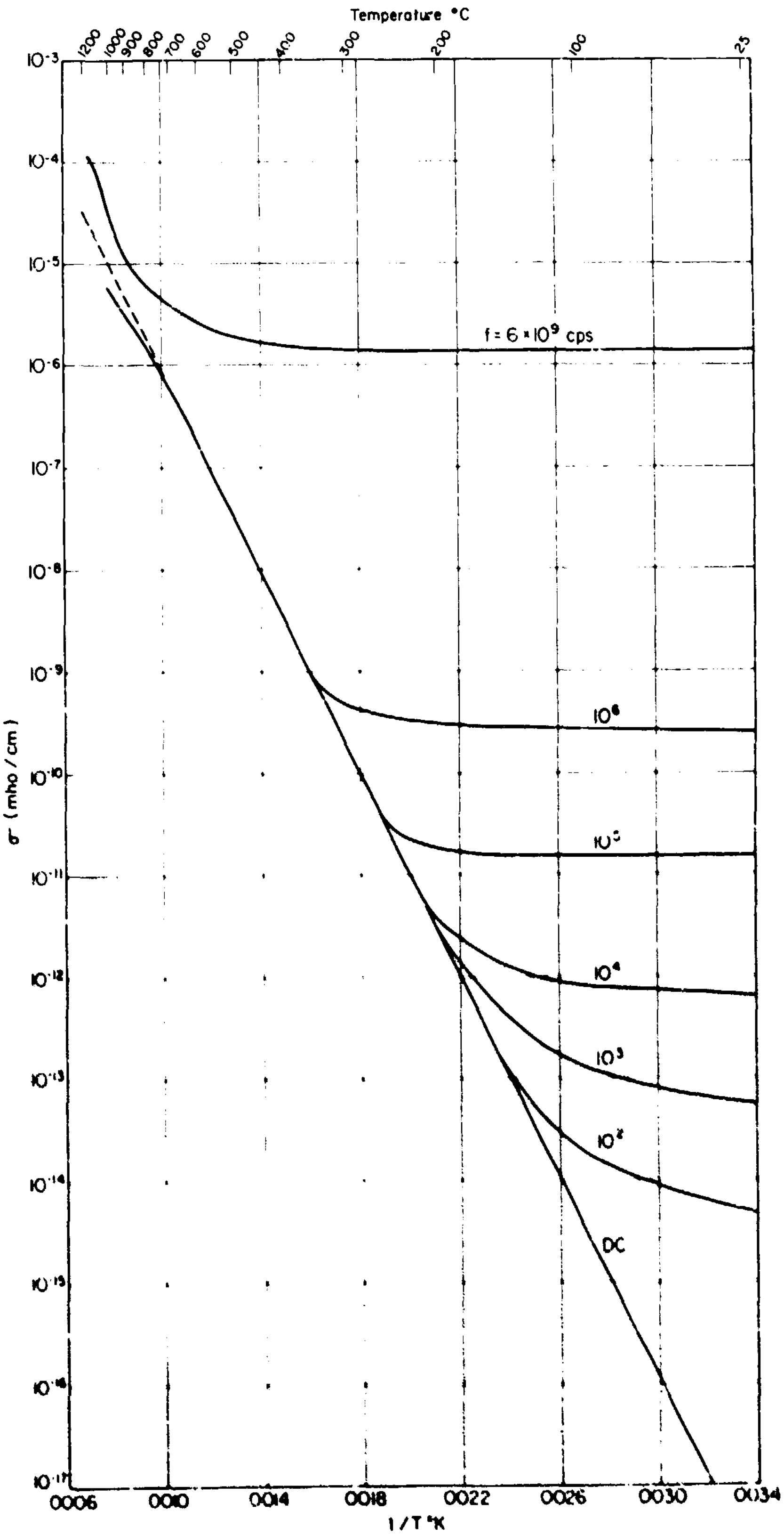
MgO ceramic, Minneapolis Honeywell Regulator Co., 99.95% MgO, density 3.52 g/cc.



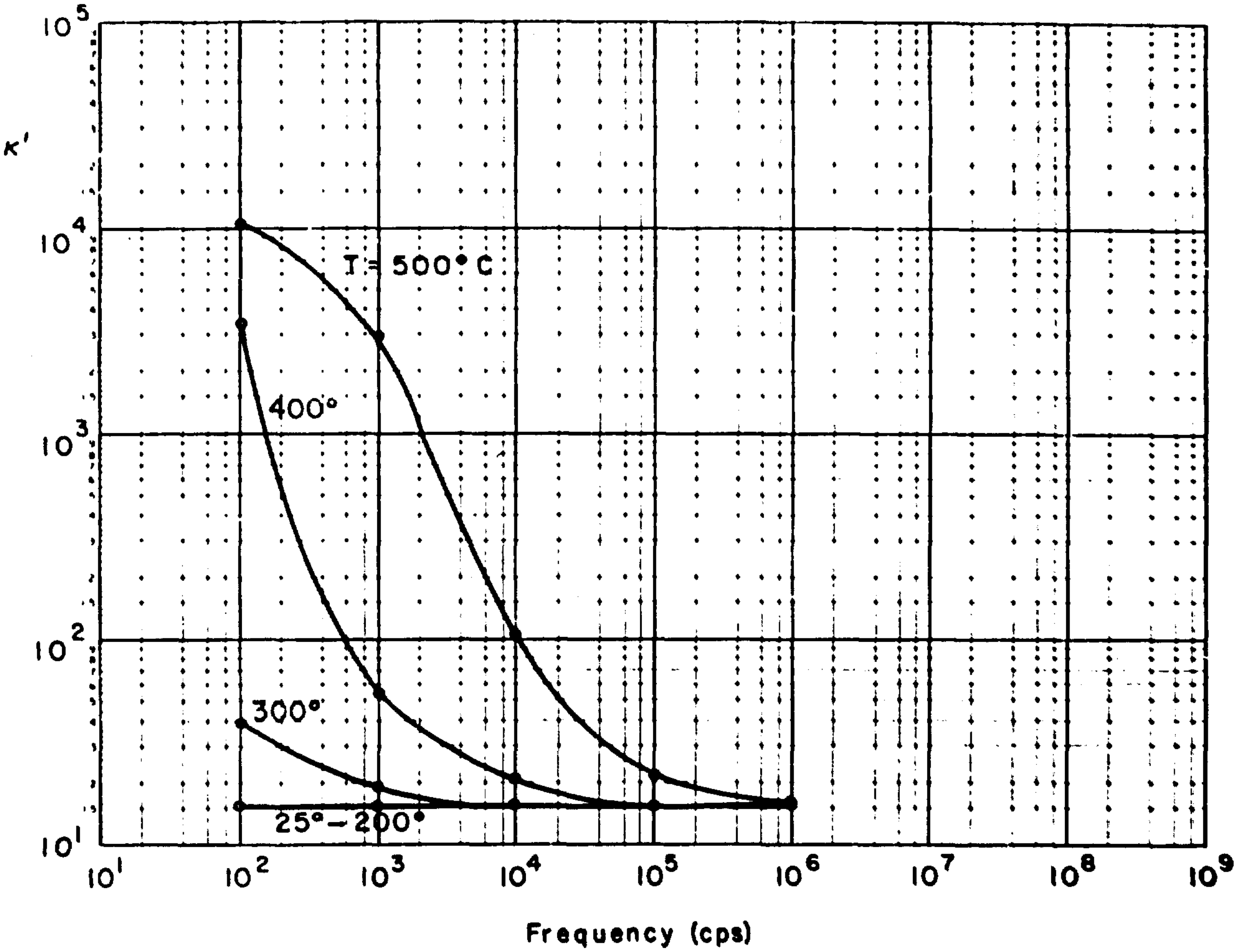

Temperature °C
1400
1200
1000
900
800
700
600
500
400
300
200
100
25
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
σ (mho/cm)
f = 3 x 10^9 cps
10^7
10^6
10^5
10^4
10^3
10^2
.0006
.0010
.0014
.0018
.0022
.0026
.0030
1/T °K

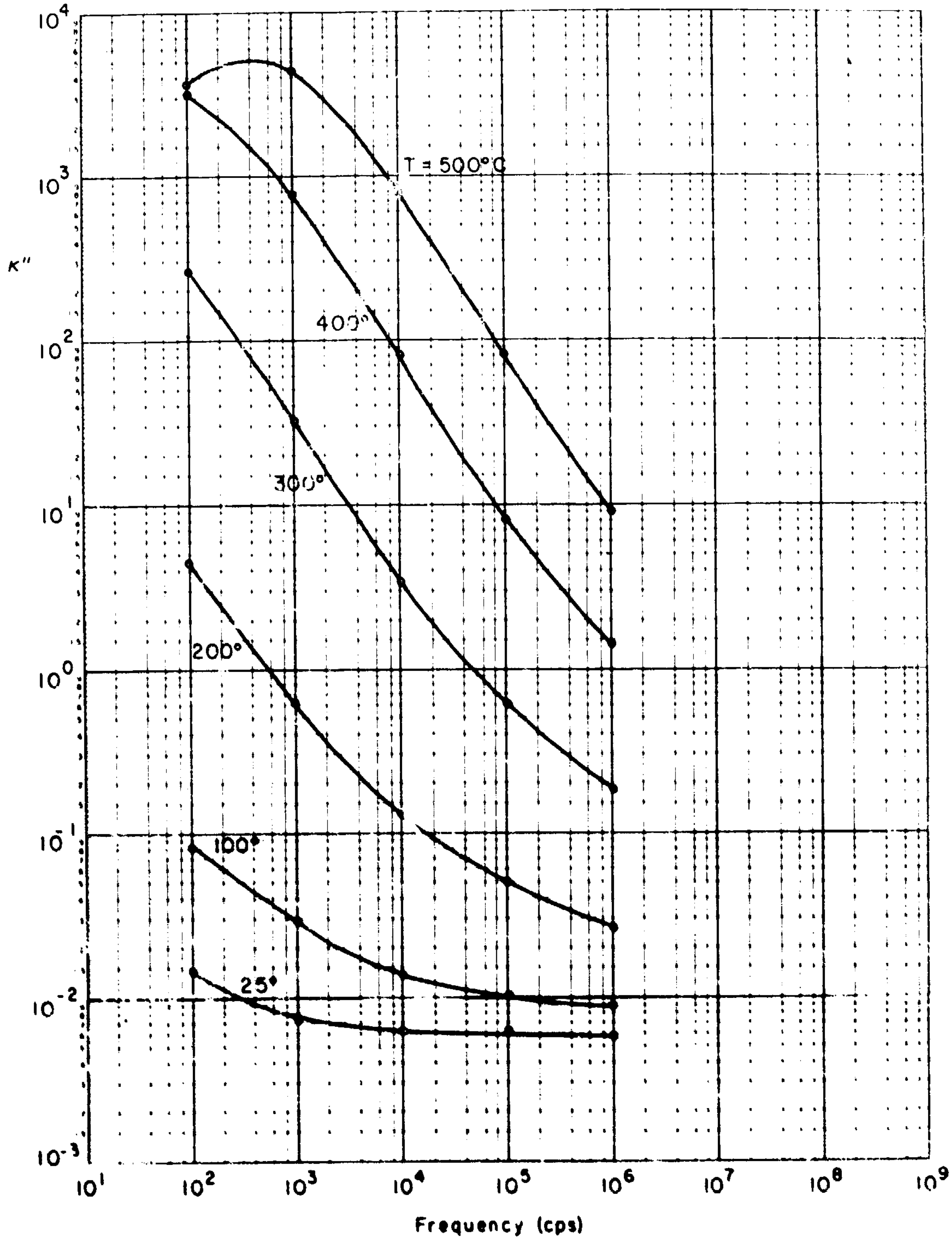
SiO_2, glass, General Electric Co., Type 101 clear, fused quartz, 99.97 to 99.98% SiO_2.




Temperature °C
1200
1000
900
800
700
600
500
400
300
200
100
25
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
σ (mho / cm)
f = 6 × 10^9 cps
10^6
10^5
10^4
10^3
10^2
DC
0006
0010
0014
0018
0022
0026
0030
0034
1 / T °K

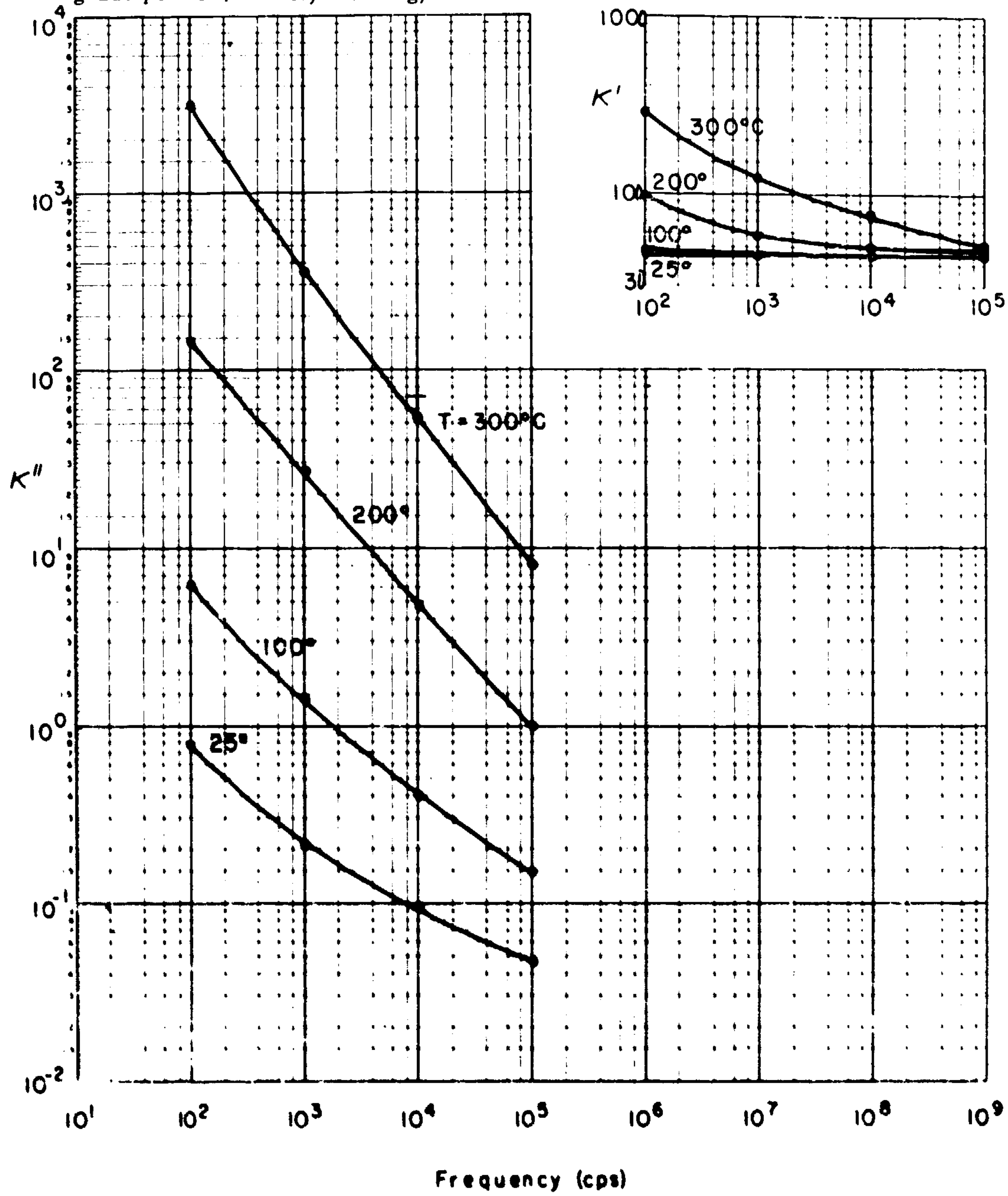
ThO_2 ceramic, Laboratory for Insulation Research; minor constituents Mg, Pb, Zn; traces of Ca, Cu, Fe, Si; density - 8.77 g/cc.



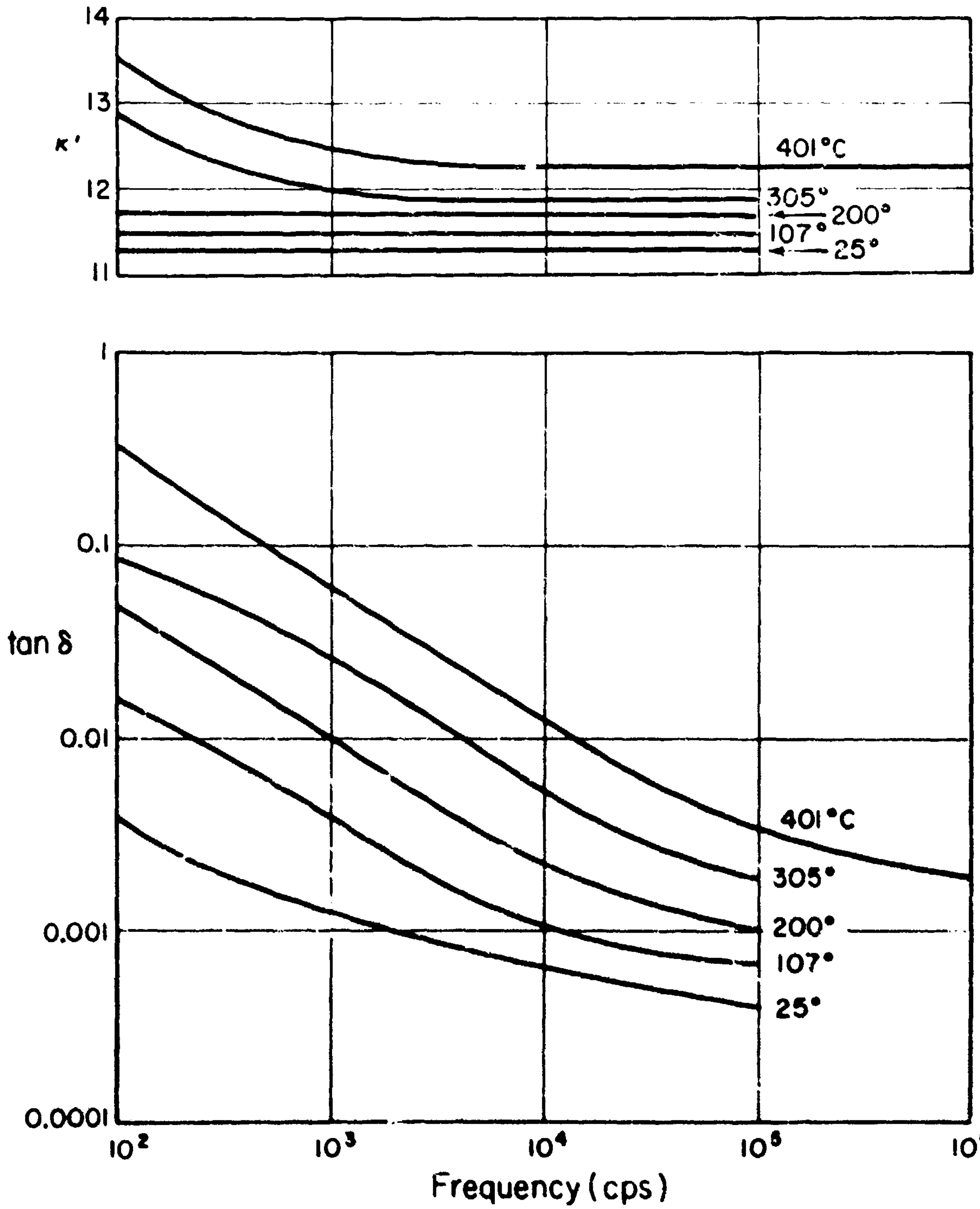


10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
κ''
T = 500°C
400°
300°
200°
100°
25°
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
Frequency (cps)

Ta_2O_5 ceramic, Laboratory for Insulation Research, hot-pressed from Ciba optical grade powder, density = 8.27 g/cc.



Y_2O_3 crystal.



III. Microwave-Cavity Data

Aluminum oxide ceramics:

Alberox A-962
American Lava AlSiMag 576
614, 96%
719, 94%
Carborundum 1542, 96%
Coors AD-99
AD-995
MC-2014
RR
Diamonite B-890-2, 90-95%
P-3142-1, 95-97%
P-3662, 85-90%
G. E. Lucalox
Minneapolis Honeywell A-203, 95%
A-127, 85%
National Beryllia Corp. Alox
Norton 99.5%
Steatit-Magnesia AG. A-18
U. S. Stoneware 610, 99%
A-212, 96%
A-216, 85%
A-312, 96+%
Std. 3050°F
Western Gold and Platinum Al-300, 97.6%
Al-400, 95%
Al-995, 99.5%
Al-1009, 99.85%

Beryllium oxide ceramics:

Brush B-6, 98.5%
B-7-6
B-7-37
F-1, 99.5%
National Beryllia, cold-pressed

Boron nitride:

Carborundum, hot-pressed
High Temperature Materials, pyrolytic

Magnesium oxide:

Minneapolis Honeywell Regulator Co.

Magnesium silicate:

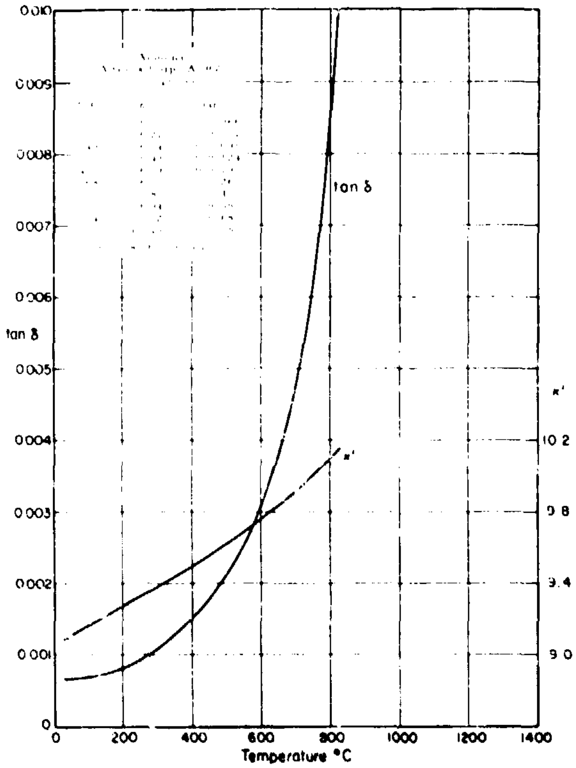
Steatit-Magnesia AG., Frequenta M

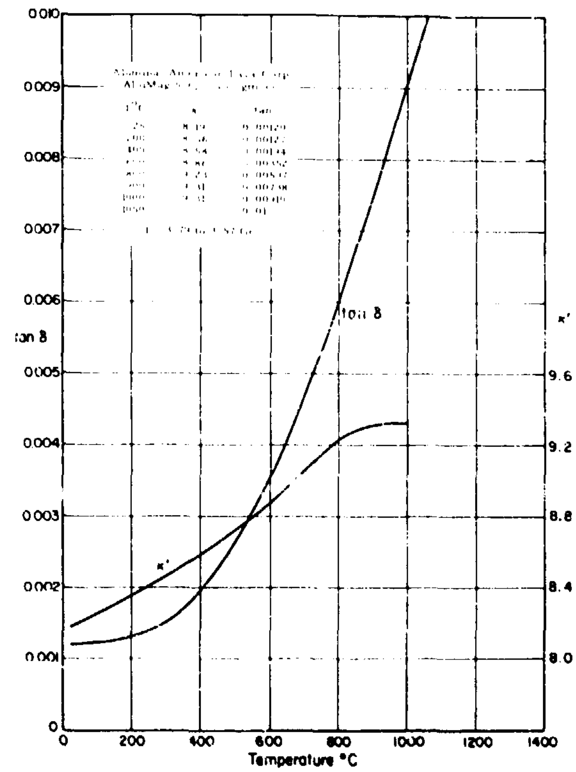
Silica:

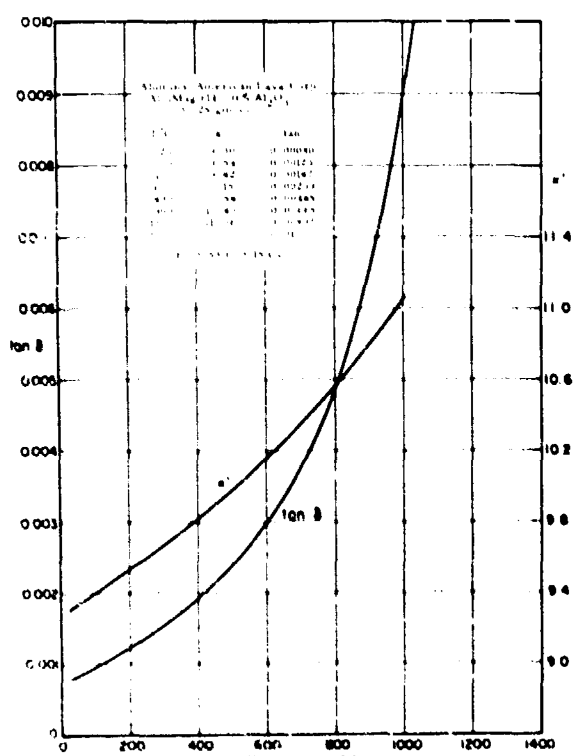
American Optical Co., Amersil, clear
Amersil, translucent
General Electric Co., 101

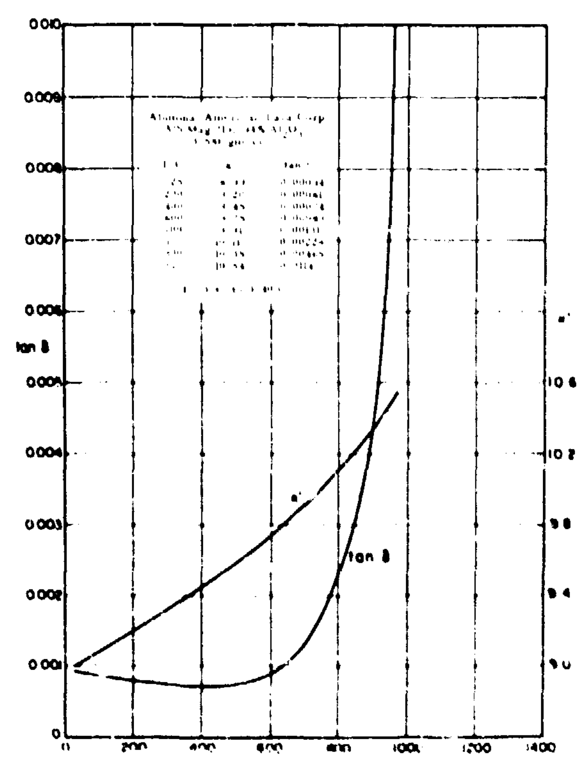
Glass ceramics:

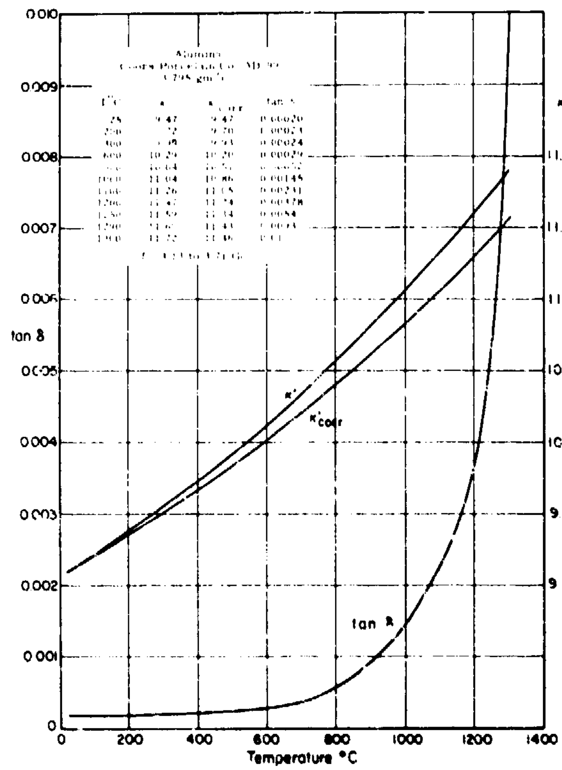
Corning 7941
9606

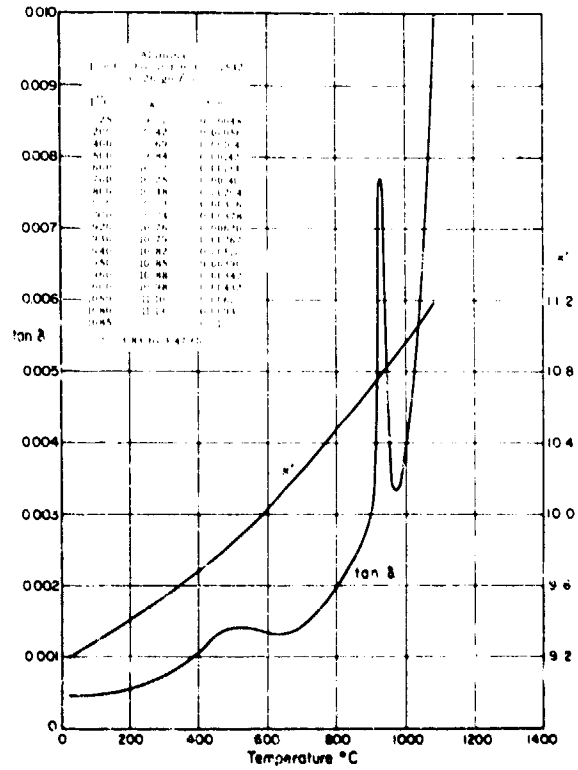

tan δ
κ'
Temperature °C
0
200
400
600
800
1000
1200
1400
9.0
9.4
9.8
10.2

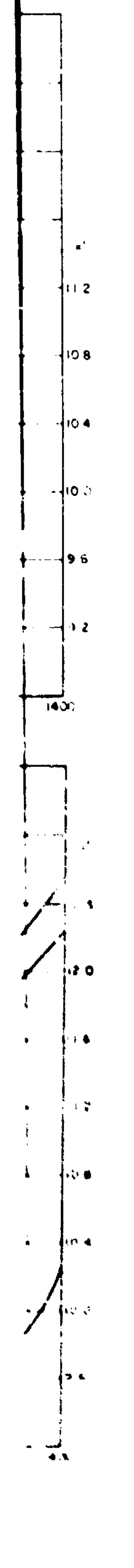

tan δ
κ'
Temperature °C
0
200
400
600
800
1000
1200
1400
8.0
8.4
8.8
9.2
9.6

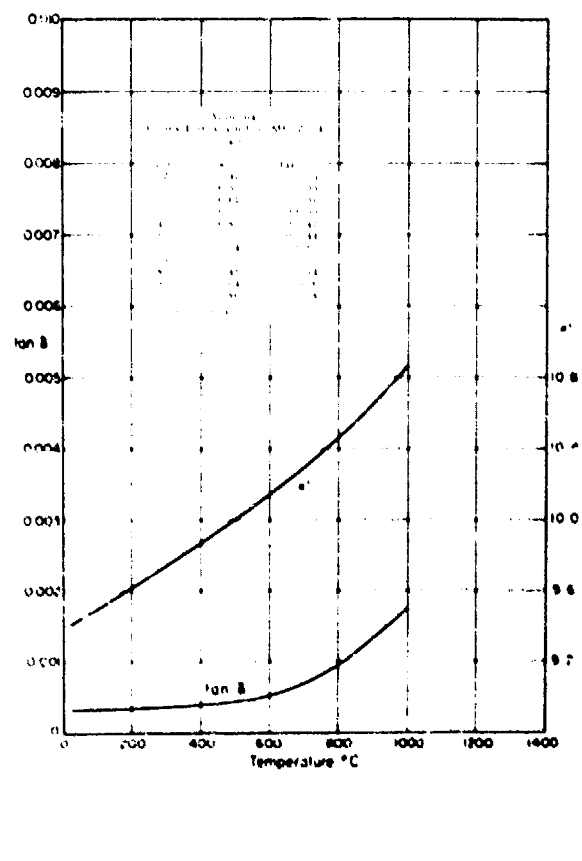

tan δ
κ'
Temperature °C
0
200
400
600
800
1000
1200
1400
9.0
9.4
9.8
10.2
10.6
11.0
11.4

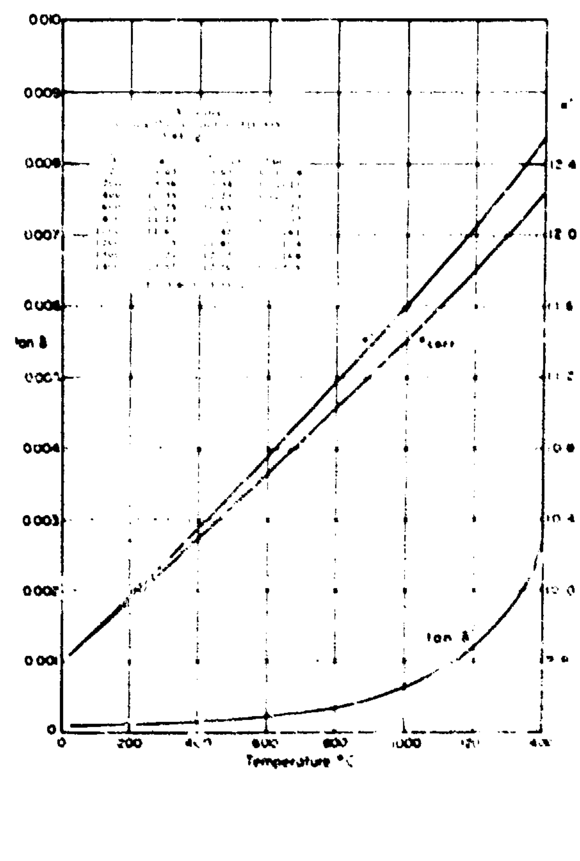

tan δ
κ'
Temperature °C
9.0
9.4
9.8
10.2
10.6

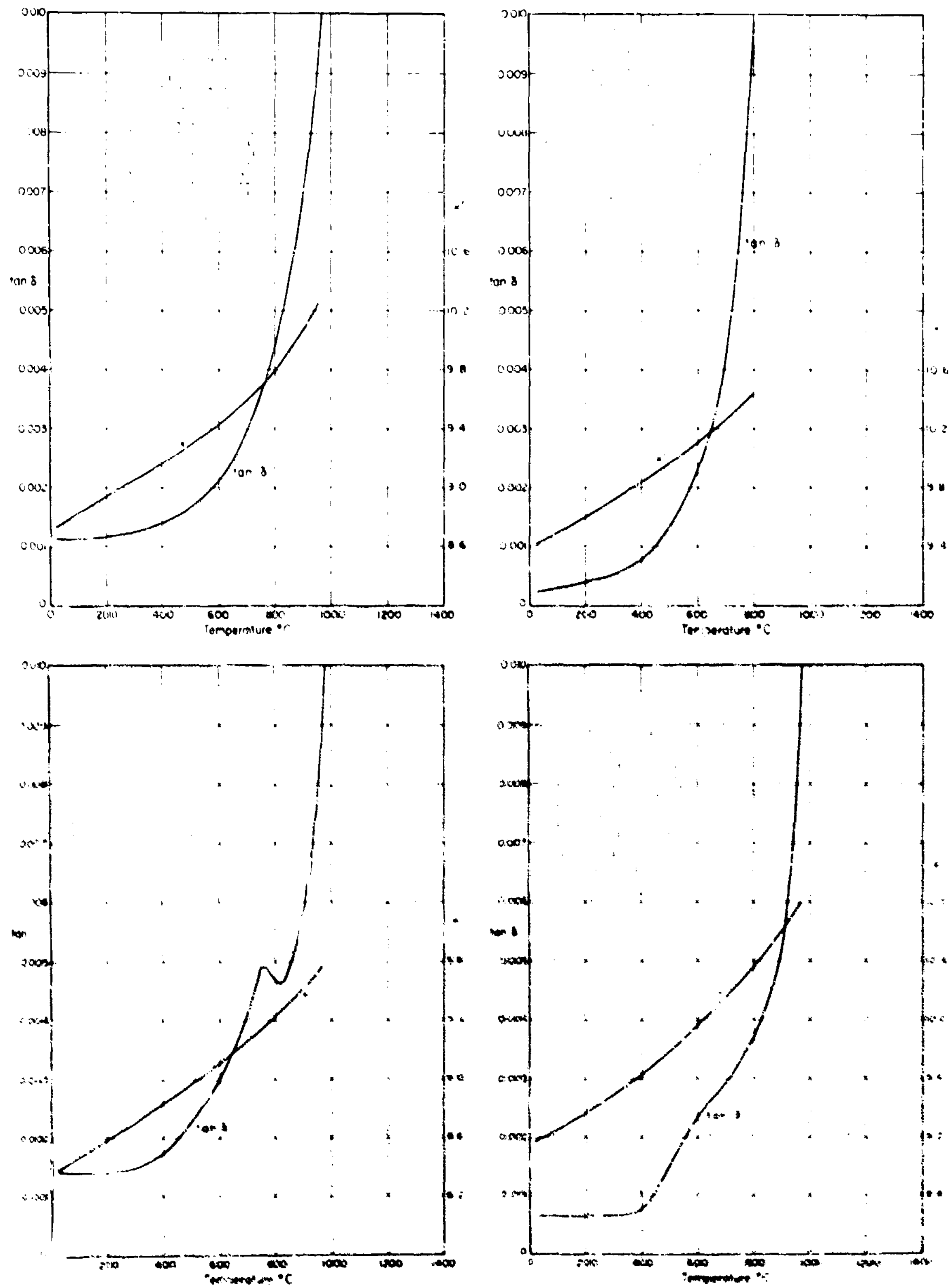

tan δ
κ'
κ'corr
tan δ
Temperature °C

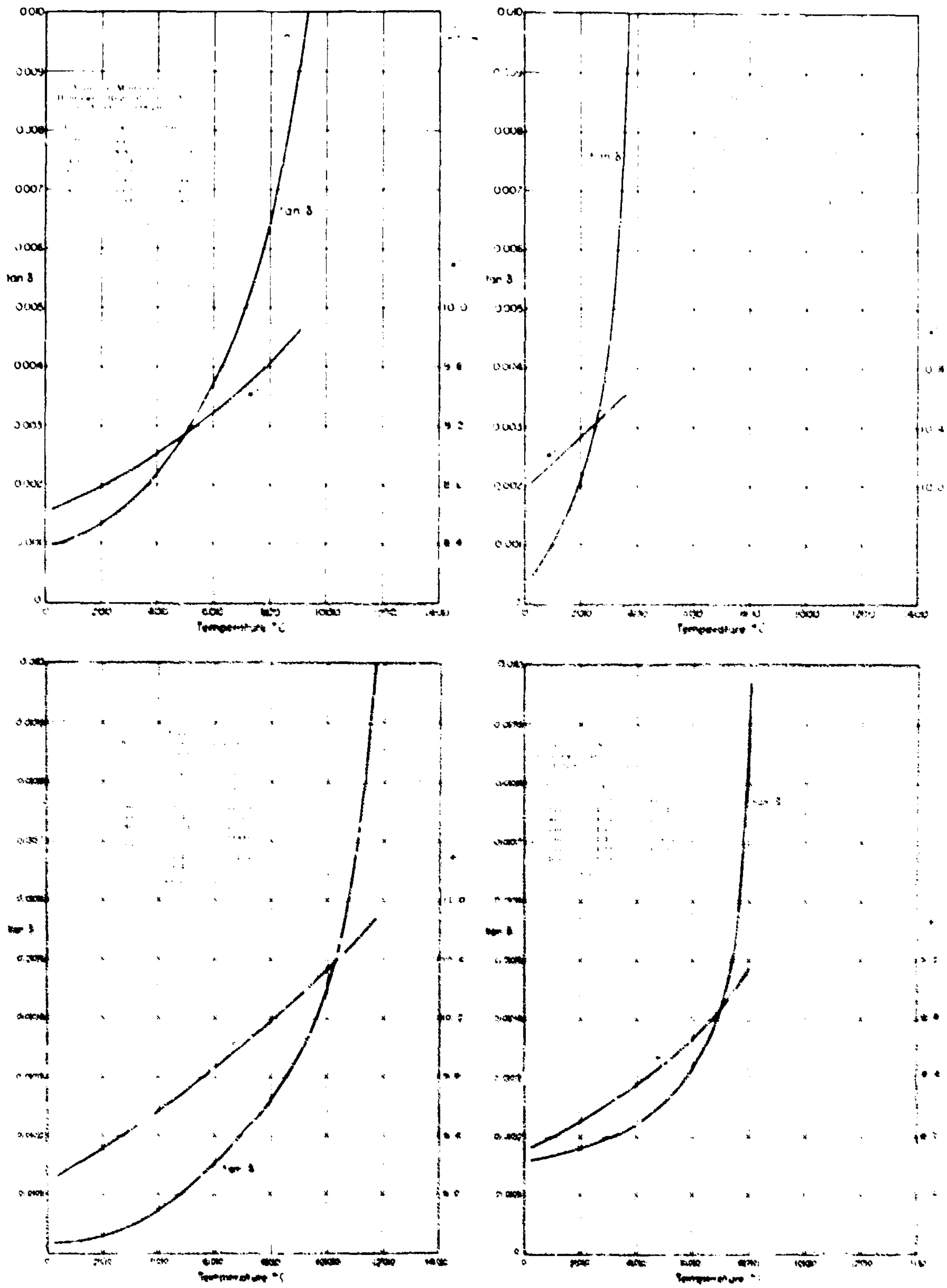

tan δ
κ'
tan δ
Temperature °C

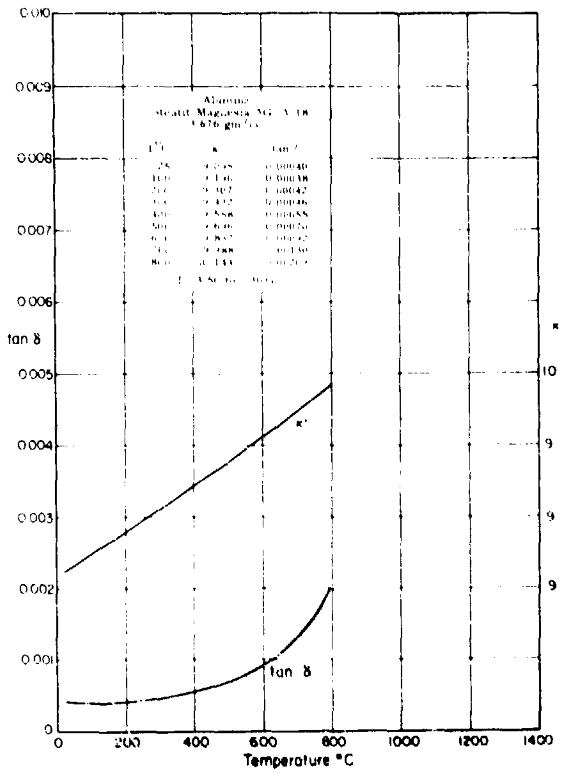


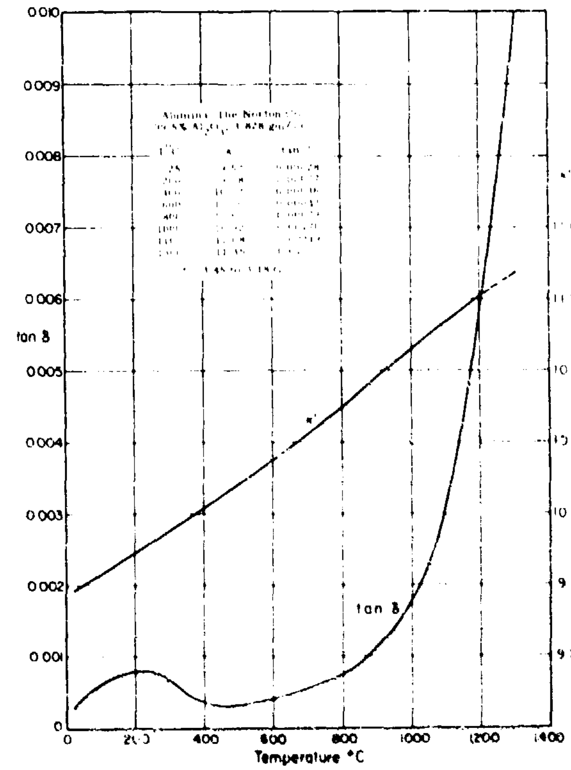

tan δ
κ'
tan δ
Temperature °C

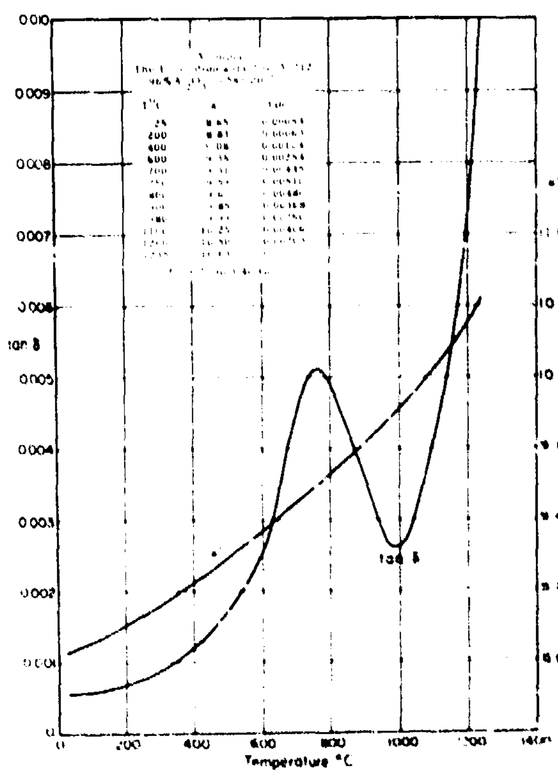

tan δ
κ'
κ'corr
tan δ
Temperature °C

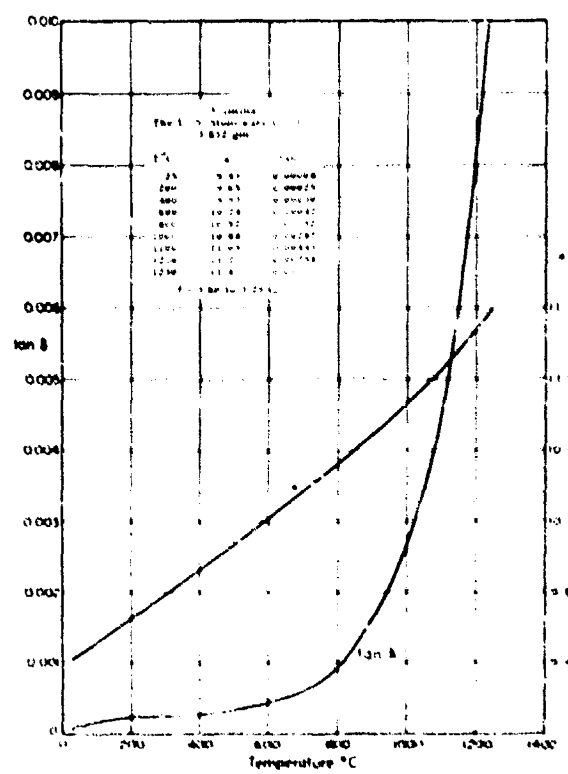

tan δ
Temperature °C
0
200
400
600
800
1000
1200
1400

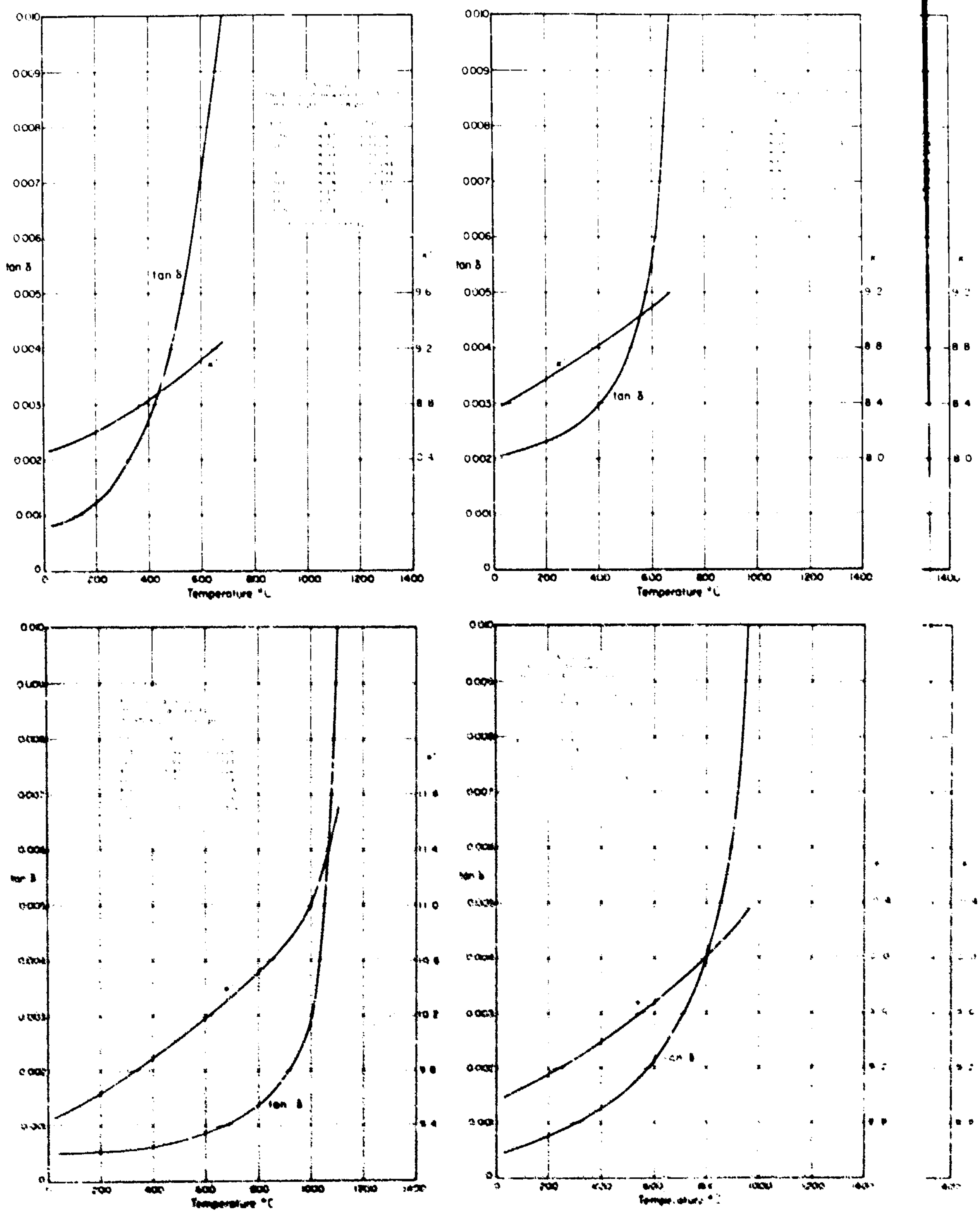

tan δ
Temperature °C
tan δ
Temperature °C
tan δ
Temperature °C
tan δ
Temperature °C

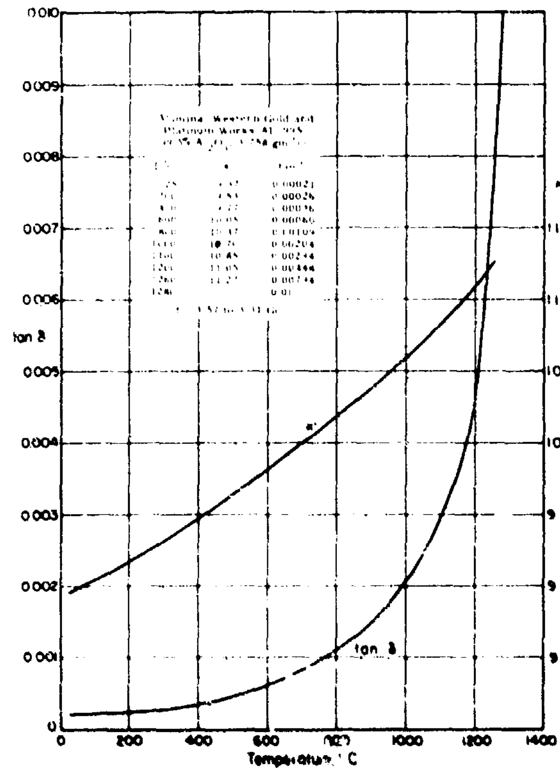

tan δ
κ'
Temperature °C

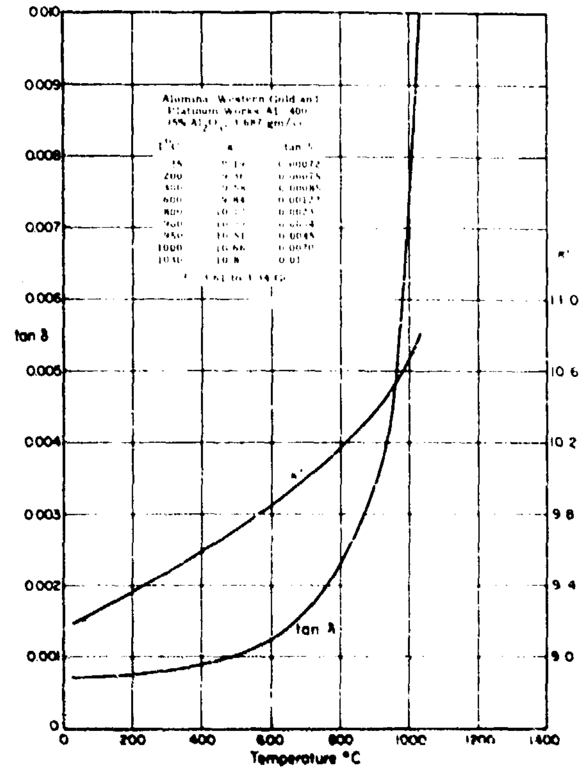

tan δ
κ'
Temperature °C

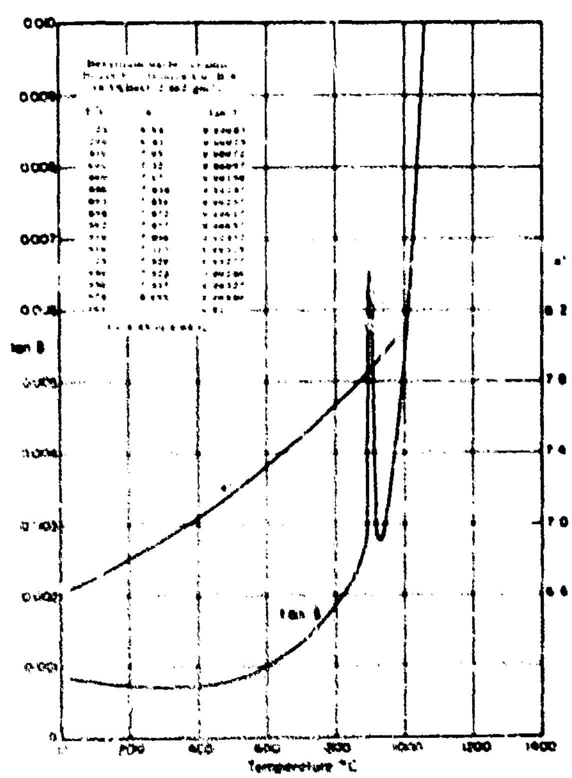

tan δ
κ'
Temperature °C

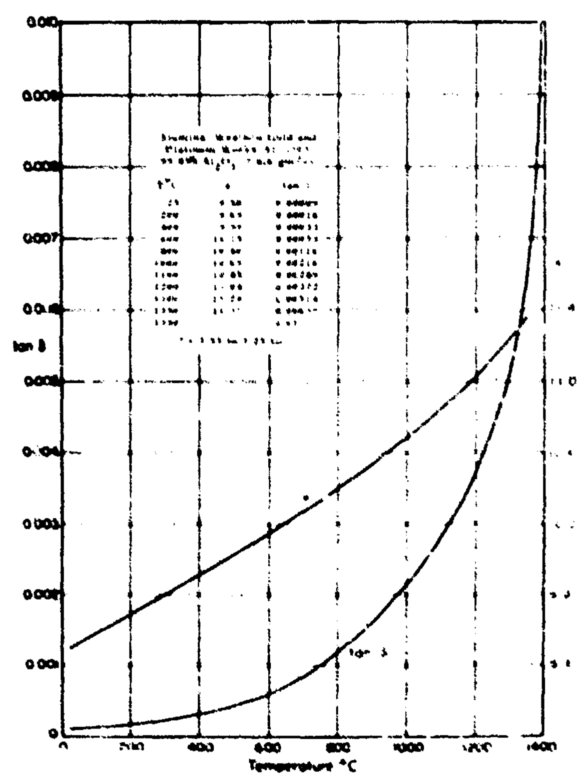

tan δ
κ'
Temperature °C

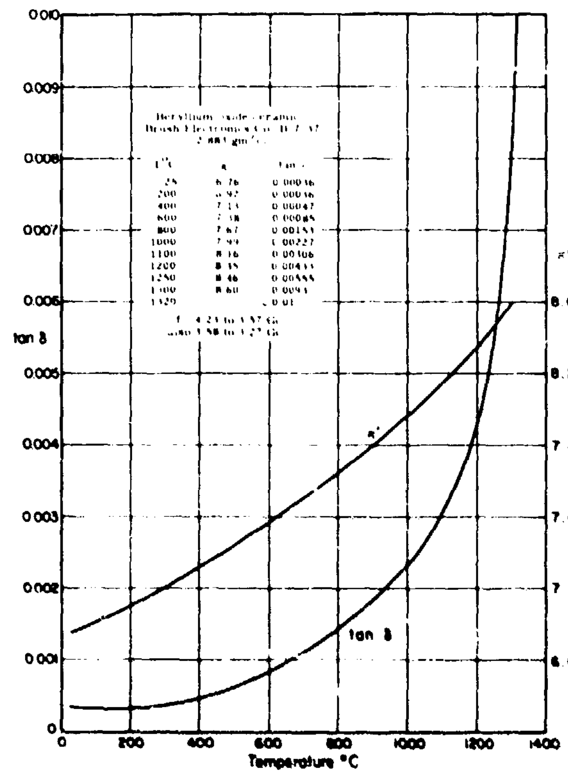

tan δ
Temperature °C
tan δ
Temperature °C
tan δ
Temperature °C
tan δ
Temperature °C

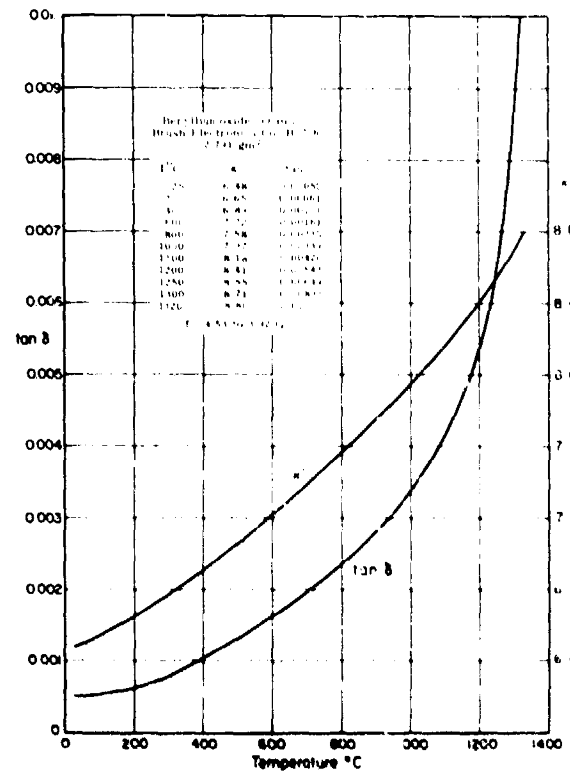

tan δ
κ'
tan δ
Temperature °C

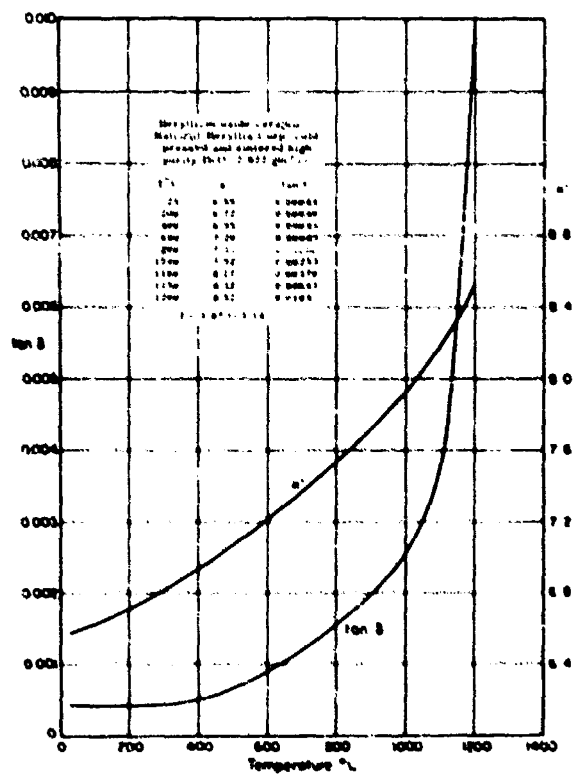

Alumina, Western Gold and Platinum Works AL 400
tan δ
κ'
tan δ
Temperature °C

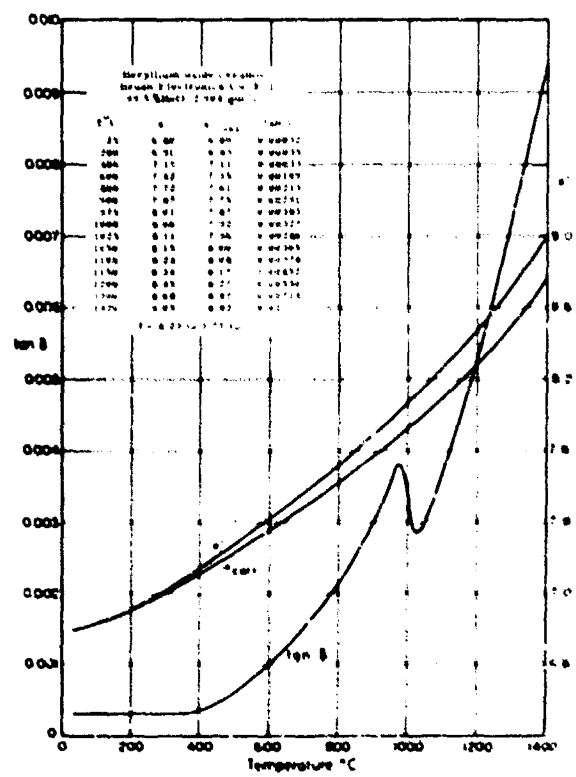

tan δ
κ'
tan δ
Temperature °C

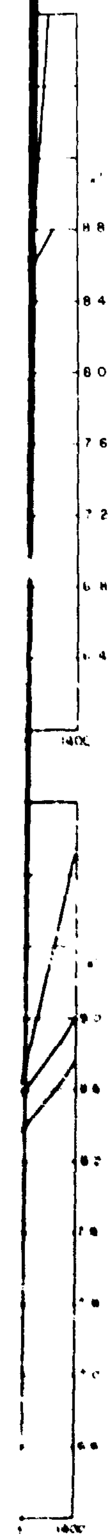

tan δ
κ'
tan δ
Temperature °C

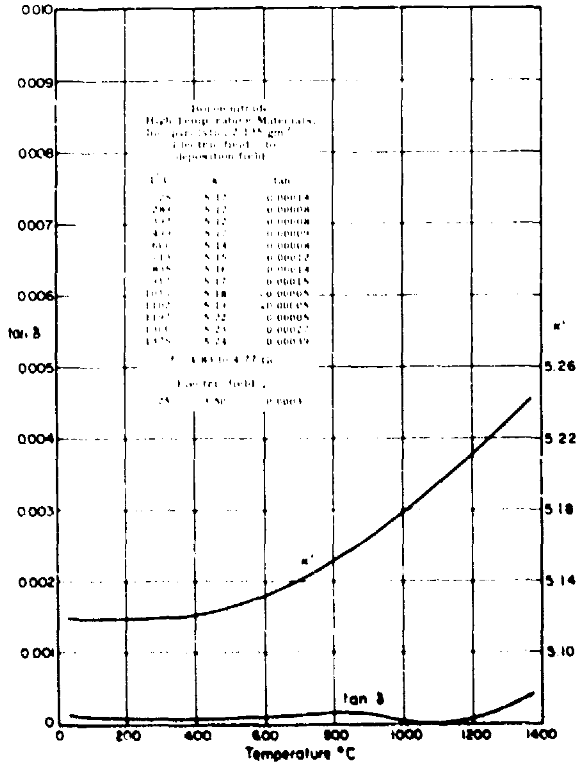

tan δ
Temperature °C

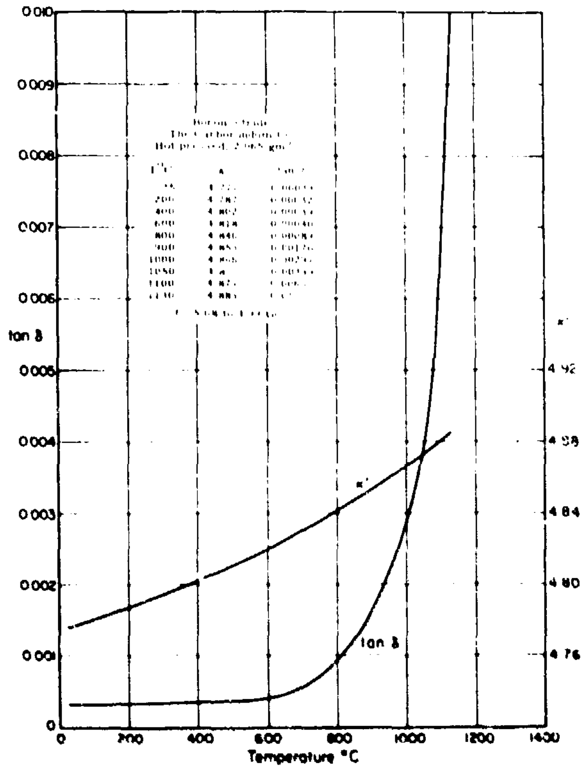

tan δ
Temperature °C

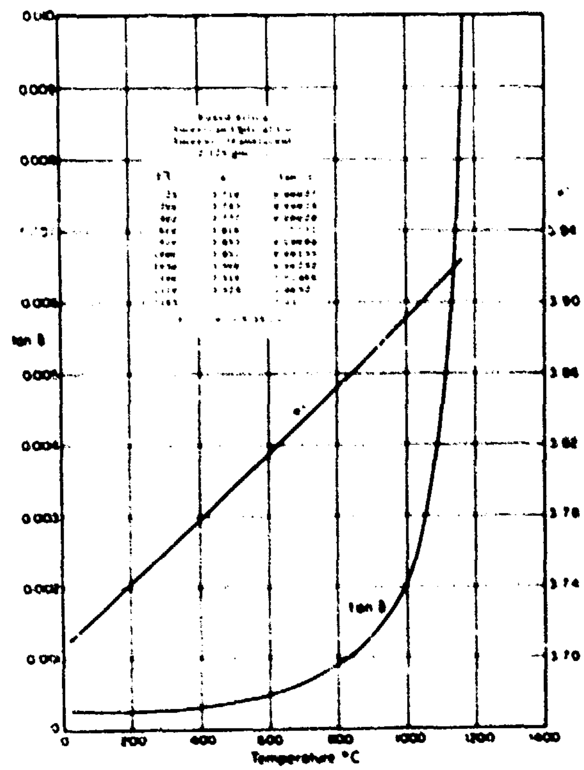

tan δ
Temperature °C

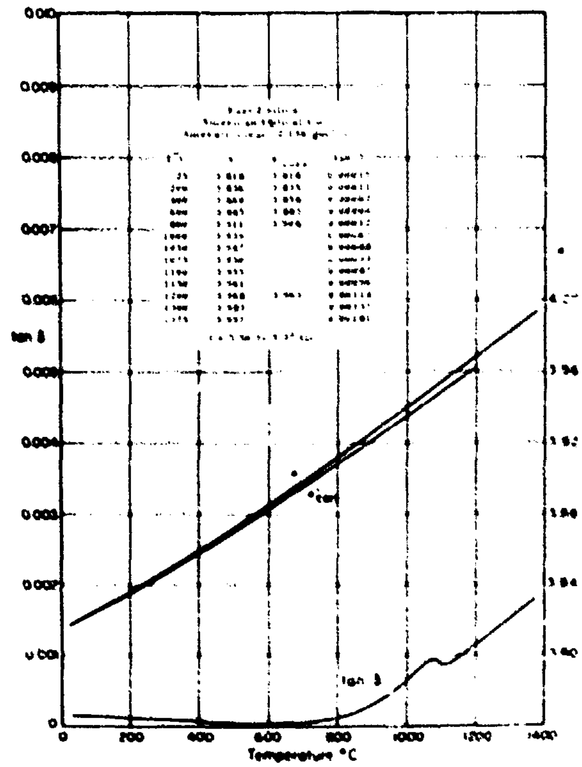

tan δ
Temperature °C

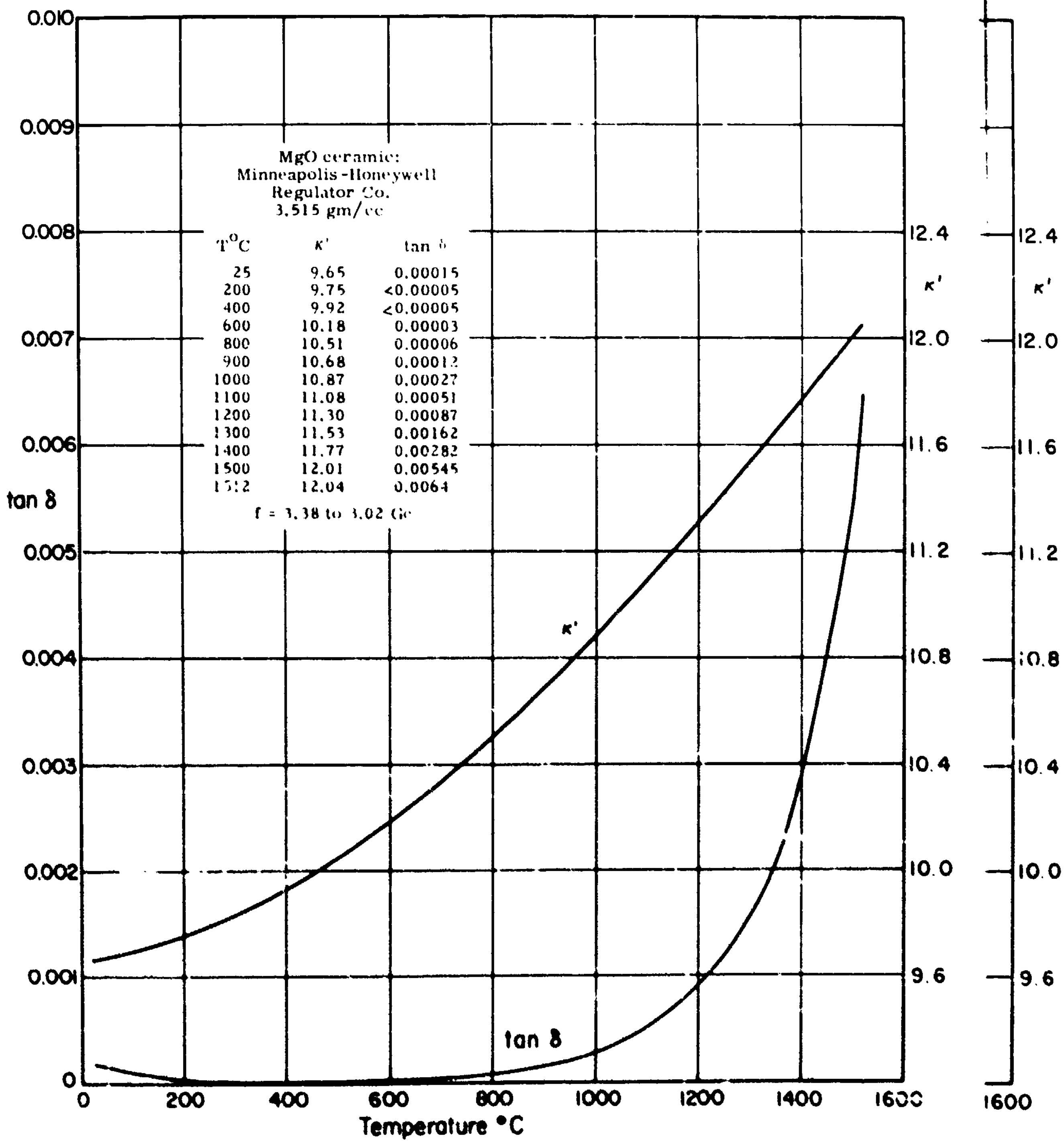



tan δ
Temperature °C
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
200
400
600
800
1000
1200
1400
5.26
5.22
5.18
5.14
5.10
tan δ

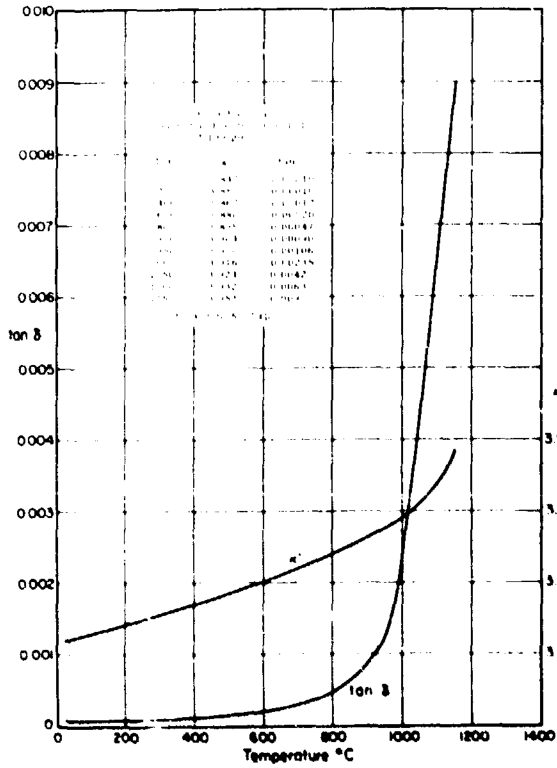

tan δ
Temperature °C
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
200
400
600
800
1000
1200
1400
4.92
4.88
4.84
4.80
4.76
tan δ

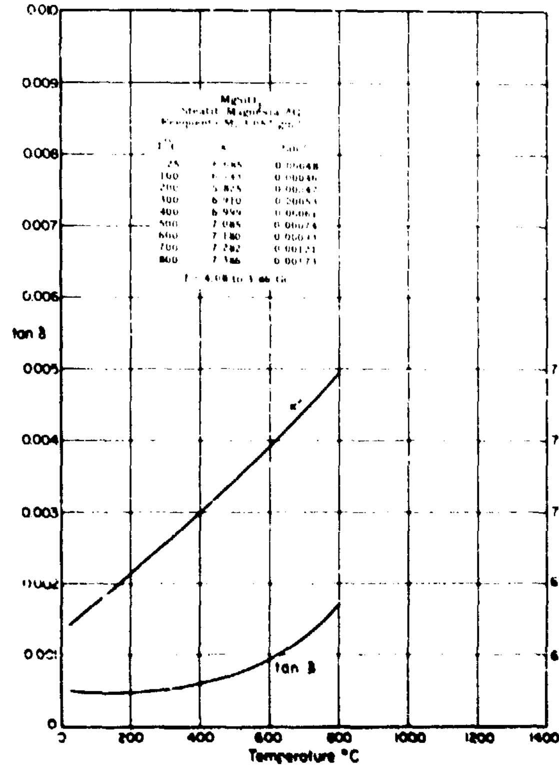

tan δ
Temperature °C
3.94
3.90
3.86
3.82
3.78
3.74
3.70
200
400
600
800
1000
1200
1400
tan δ

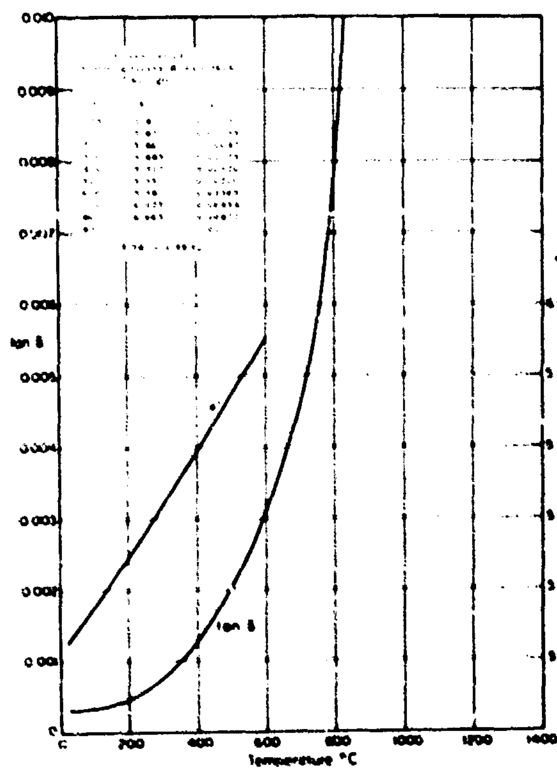

tan δ
Temperature °C
0.010
0
200
400
600
800
1000
1200
1400
3.96
3.84
3.80
tan δ

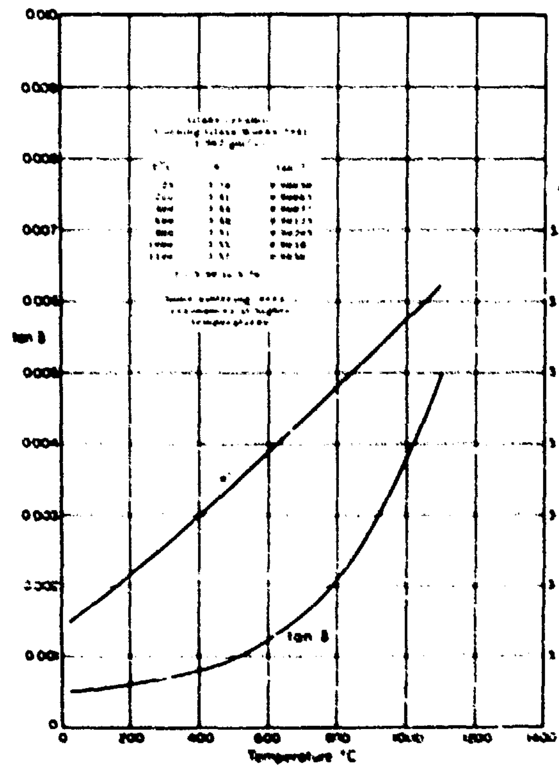


T°C	κ'	tan δ
25	9.65	0.00015
200	9.75	<0.00005
400	9.92	<0.00005
600	10.18	0.00003
800	10.51	0.00006
900	10.68	0.00012
1000	10.87	0.00027
1100	11.08	0.00051
1200	11.30	0.00087
1300	11.53	0.00162
1400	11.77	0.00282
1500	12.01	0.00545
15[illegible]2	12.04	0.0064


tan δ
ε'
Temperature °C
0 200 400 600 800 1000 1200 1400
0.001 0.002 0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010
3.84 3.88 3.92 3.96


tan δ
ε'
Temperature °C
0 200 400 600 800 1000 1200 1400
0.003 0.004 0.005 0.006 0.007 0.008 0.009 0.010
6.6 6.8 7.0 7.2 7.4


tan δ
ε'
Temperature °C
200 400 600 800 1000 1400
3.84 3.88 3.92 3.96 4.00


tan δ
ε'
Temperature °C
0 200 400 600
0.001 0.003 0.005 0.006 0.007 0.008 0.010
3.36 3.40 3.44 3.48 3.52 3.56 3.60

IV. Summary

Measurement Results

As the tabulations have shown, the majority of samples showed dielectric losses increasing steadily with temperature, and the highest purity samples had lowest losses in a general sense. On the low-loss samples we are accumulating lower-frequency measurements which show that high-temperature microwave losses are not due to the same mechanisms as low-frequency losses, i.e., effective transconductance due to motion of electrons and ions with the electric field. In typical cases such as shown for G. E. fused quartz and Coors AD-99, the low-frequency transconductance accounts for 1/10 to 1/5 of the microwave loss. Yet it is generally true that adding impurities increases both the low-frequency conductance and the microwave loss. This relation means that the impurities (and their associated dislocations and lattice imperfections) that cause low-frequency charge transfer also lower the melting point and extend infrared vibration losses to lower frequencies. Two of the materials, Brush B-6 beryllia and Carborundum alumina, have sharp absorptions which look like vibration spectra. Exact interpretation will depend on data taken versus frequency at fixed temperatures.

None of the samples, except for Corning 7941, showed appreciable room-temperature changes. Thus platinum contamination was not a problem during these runs, which lasted 4 to 12 hours with usually more than 3/4 of the time above 700°C.

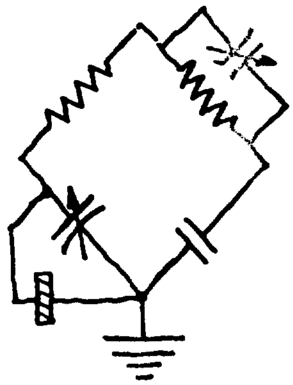
Acknowledgments

Mr. Walter Webber of Sylvania Electric Products, Waltham, Mass., collected many of the samples for microwave measurements, and his company paid part of the measurement costs. Sample-holder design and construction were made by R. E. Charles. Part of the low-frequency measurements and most of the data figures were made by Mrs. B. B. East. The excellent drawings of sample holders are the work of J. J. Mara.

APPENDIX A

Calculations for Lumped Circuits

Zone I. Capacitance substitution method



Room-temperature measurements in micrometer holder:

C'' - All-out capacitance balance.

C' - Capacitance balance with empty sample holder; gap at sample thickness.

D' - Dissipation-factor reading, empty holder.

C_x - Capacitance balance with sample.

D_x - Dissipation-factor balance with sample.

C_o - Geometric capacitance of sample = $\epsilon_o(A/t)$

= 0.8854(A/t) pf/cm; 0.06954 (dia.)^2 pf/cm.

$$C_s = C' - C_x + C_o \; ; \tag{A-1}$$

$$\kappa' = \frac{C_s}{C_o} \; ; \tag{A-2}$$

$$\tan \delta_s = \frac{C''}{C_s} \frac{D_x - D'}{100} \frac{f}{f_o} \, , \tag{A-3}$$

where f is the operating frequency, f_o the frequency for which dissipation factor dial is direct reading.

Measurement in high-temperature holder:

C'' - All-out capacitance balance.

C' - Capacitance balance, sample switched out.

D' - Dissipation balance, sample switched out.

C_x - Capacitance balance, sample in.

D_x - Dissipation balance, sample in.

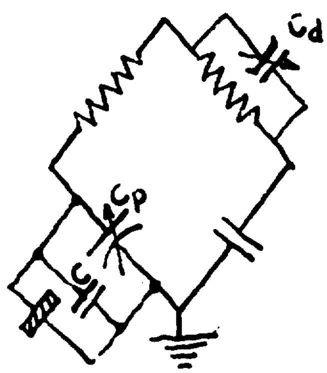
$$C_h = C' - C_x - C_s, \text{ by definition;} \qquad (A-4)$$

$$C_s = C' - C_x - C_h - \left[\frac{C'' \cdot \Delta D \cdot D}{1 + D^2} = \frac{C''}{100} K_2 \text{ (for D' close to zero)} \right] ;(A-5)$$

$$\tan \delta_s = \frac{C''}{C_s} \; \frac{\Delta D}{1 + D^2} = \frac{C''}{C_s} \Delta D K_1 \, , \qquad (A-6)$$

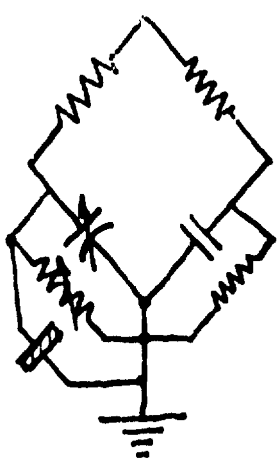
where $\Delta D = \frac{D_x - D'}{100} \cdot \frac{f}{f_o}$; $D = \left(\frac{D_x}{100} + 0.034 \right) \frac{f}{f_o}$; and K_1 and K_2 are as shown in Figs. A-1, A-2, and A-3.

Zone II. Capacitance substitution method with added known parallel capacitor C (to reduce over-all loss tangent)



Eq's. 5 and 6 apply, but with $C'' = C + C_p$, with sample and holder out.

Zone III. Resistance-capacitance substitution



R_x - Resistance balance, sample in.

C_x - Capacitance balance, sample in.

R' - Resistance balance, sample out.

C' - Capacitance balance, sample out.

$$C_s = C' - C_x - C_h \; ;$$

$$\tan \delta_s = \frac{(R_x - R')}{\omega C_s R_x R'} \, .$$

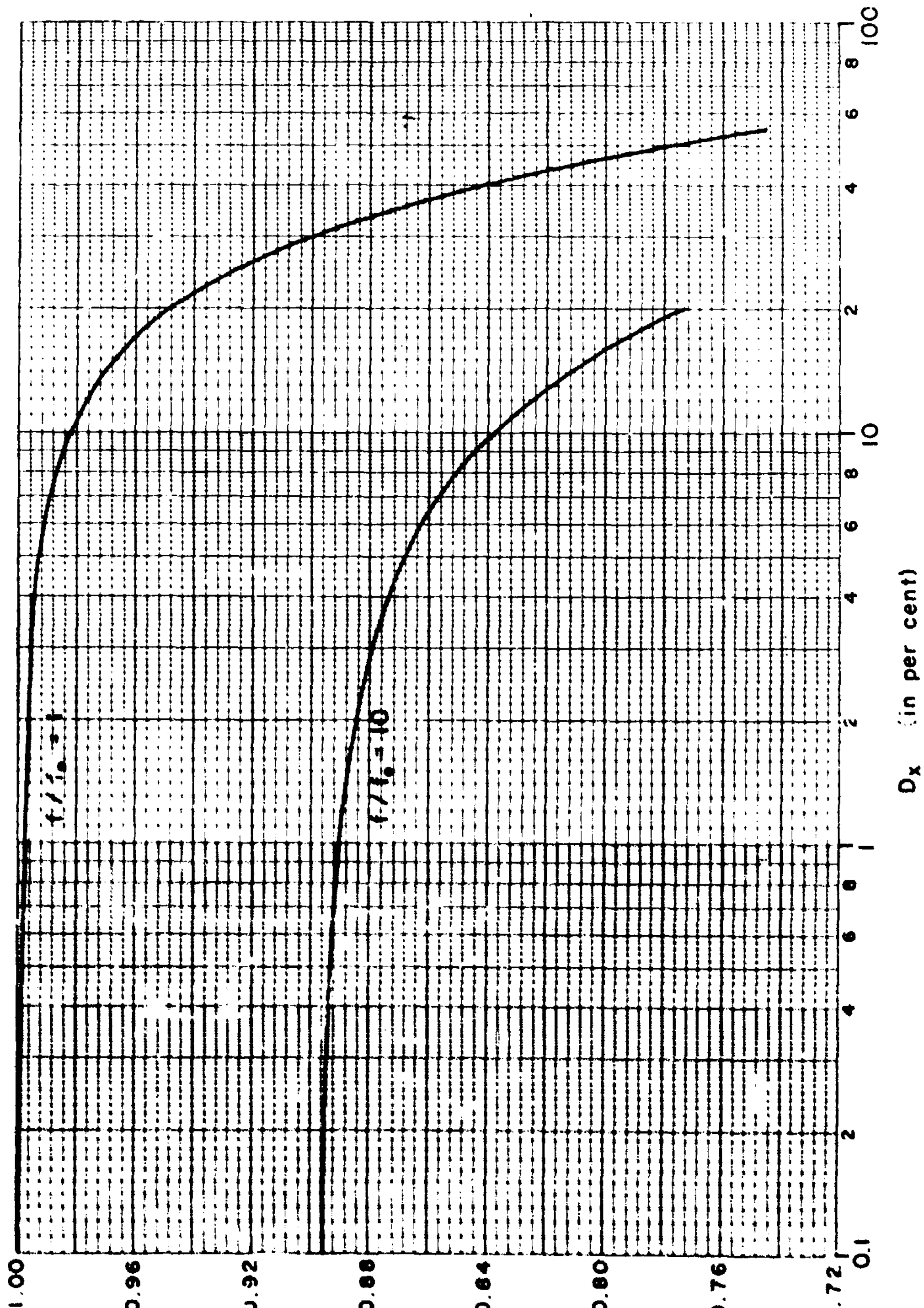


Fig. A-1. Loss-tangent correction factor K_1 for G. R. 716-C bridge.

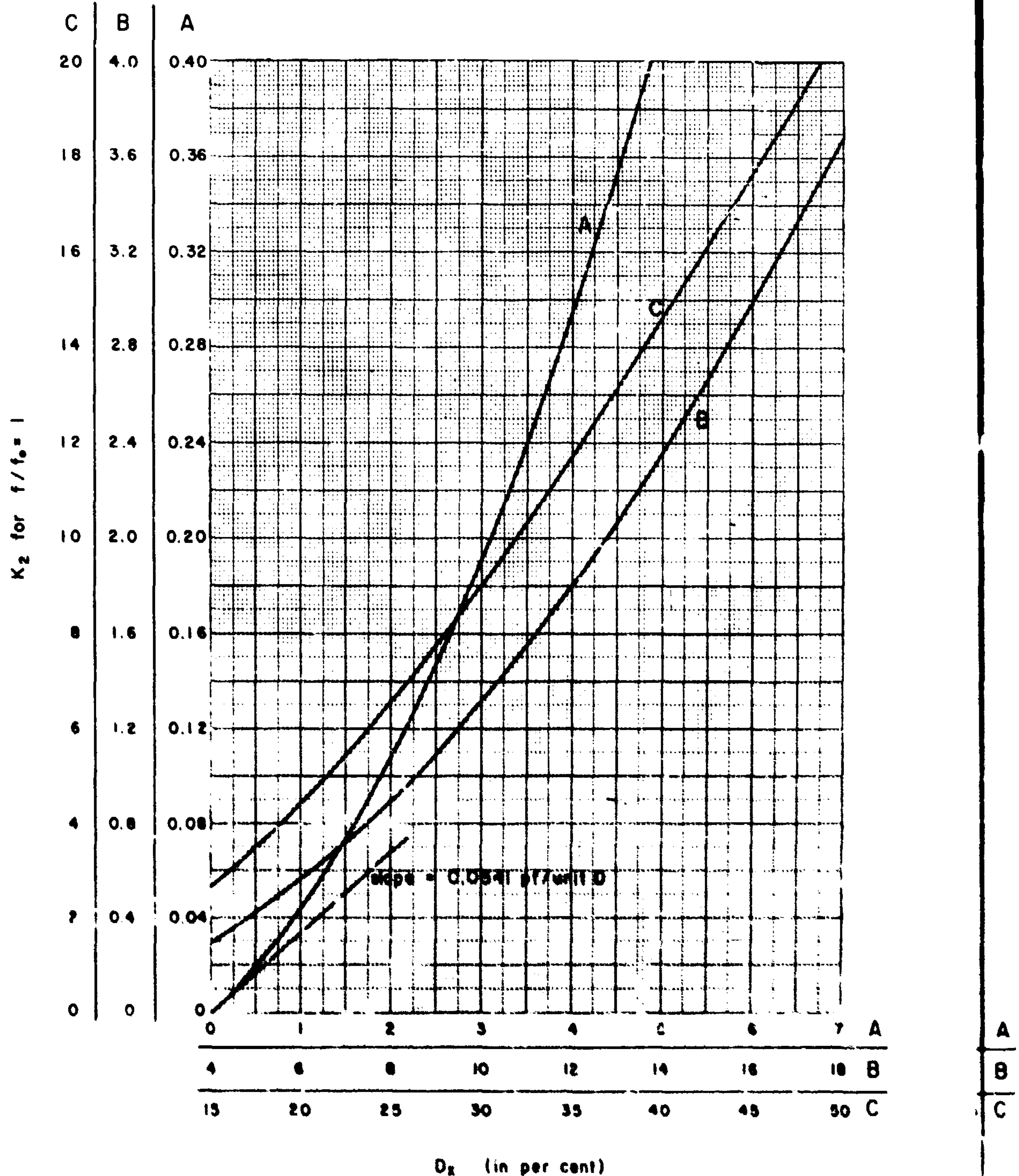


Fig. A-2. Capacitance-correction factor K_2 for G. R. 716-C bridge, $f/f_o = 1$

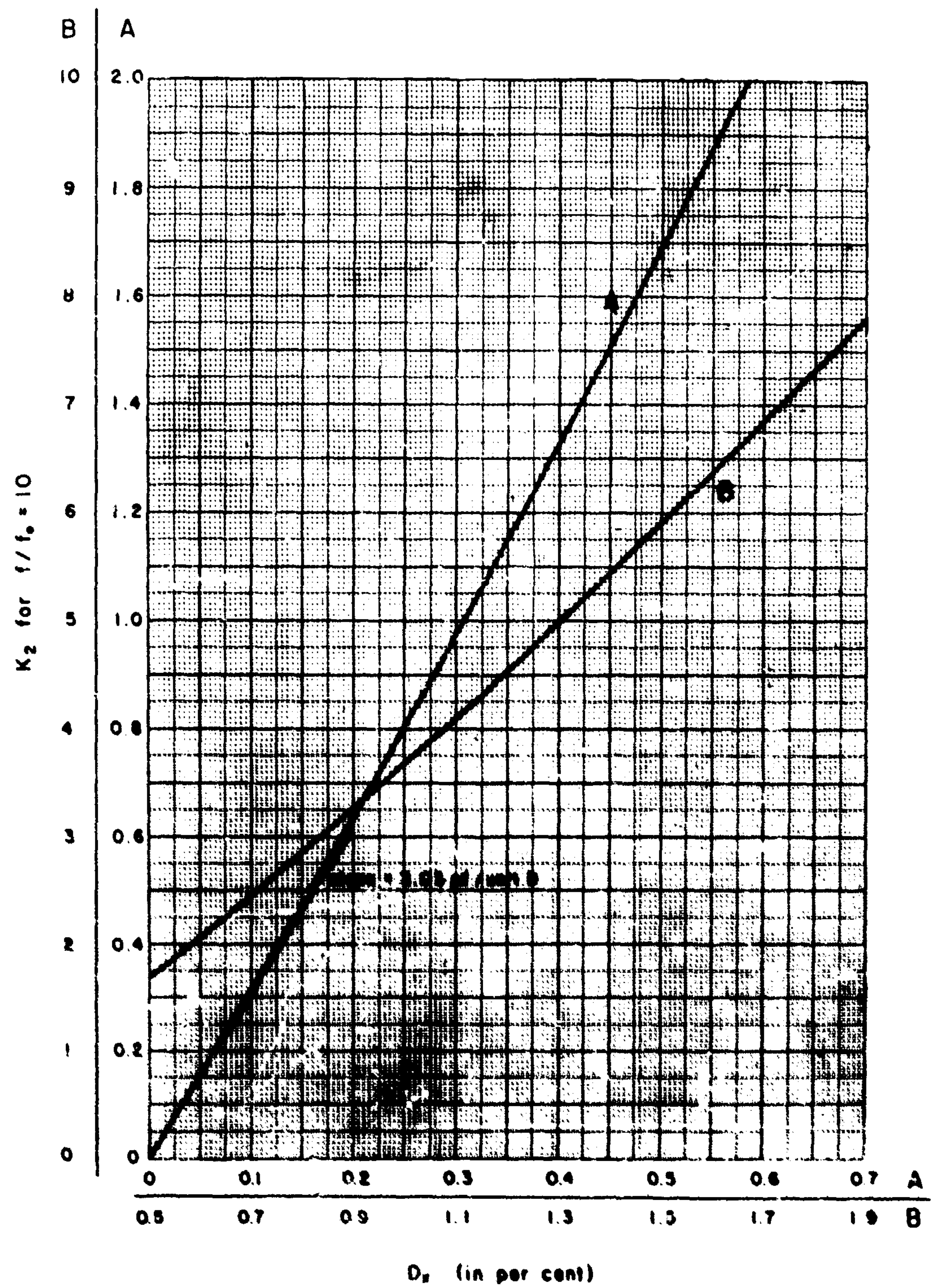
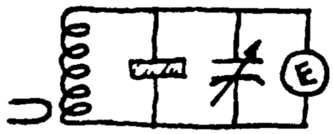
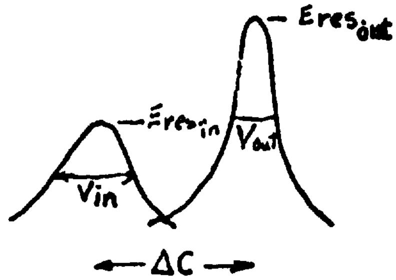


Fig. A-3 Capacitance-correction factor K_2 for G.R. 716-C bridge. $f/f_o = 10$.

Zone IV. Resonant circuit reentrant cavity with lumped sample





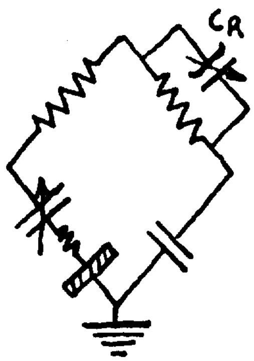
Micrometer holder $C_s = \Delta C + C_o$

High-temperature holder $C_s = \Delta C - C_h$

$$\tan \delta = \frac{\Delta V}{2C_s\left[\left(\frac{E_{res.}}{E_{off\ res.}}\right)^2 - 1\right]^{1/2}},$$

$$\tan \delta = \frac{V_{out}}{2C_s\left[\left(\frac{E_{res.}}{E_{off\ res.}}\right)^2 - 1\right]^{1/2}}\left(\frac{E_{res.\ out}}{E_{res.\ in}} - 1\right).$$

Zone V. Series Schering bridge



ΔX - Change in reactance balance when bridge terminals are shorted.

ΔR - Change in resistance balance when bridge terminals are shorted.

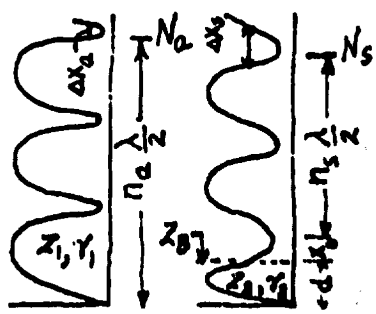
$$\tan \delta_T = \frac{\Delta R}{\Delta X},$$

$$C_s = \frac{1}{\omega \Delta X(1 + \tan^2 \delta)} - C_h,$$

$$\tan \delta_s = \frac{C_s + C_h}{C_s} \tan \delta_T$$

APPENDIX B.

Zone VI. Standing-wave method



General relations, sample at short:

$$X_o = N_s + n_a\left(\frac{\lambda}{2}\right) - n_s\left(\frac{\lambda}{2}\right) - d,$$

$$\Delta x = \Delta X_s - \left[X_o + n_s\left(\frac{\lambda}{2}\right)\right]\frac{\Delta X_a}{n_a\left(\frac{\lambda}{2}\right)}.$$

$$\frac{E_{min}}{E_{max}} = \frac{\pi \Delta X}{\lambda} \text{ (see Table B-1).}$$

$$\frac{Z_B}{Z_1} = \frac{\dfrac{E_{min}}{E_{max}} - j \tan \dfrac{2\pi x_o}{\lambda}}{1 - j \dfrac{E_{min}}{E_{max}} \cdot \tan \dfrac{2\pi x_o}{\lambda}} .$$

For TE waves, nonmagnetic sample,

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1} \frac{Z_B}{Z_1}$$

$\gamma_2 d$ is determined from charts or tables of tanh x/x

$$\kappa' = \frac{u + \left(\frac{\lambda}{2\pi d} \cdot \gamma_2 d\right)^2}{1 + u} ,$$

where x_o is the distance of first minimum in standing wave above sample, Δx width of minimum at twice minimum power points, Z_1 intrinsic impedance of air-filled guide, Z_2 intrinsic impedance of sample-filled guide, γ_1 propagation function for air-filled guide, γ_2 propagation function for sample-filled guide, λ wavelength in air-filled guide, $u = \lambda^2/\lambda_c^2$, λ_c = 3.412586 times the radius.

Simplified calculations (for $\tan \delta < 0.1$ and $n\kappa'' < 0.2$, where n is number of wavelengths in sample)

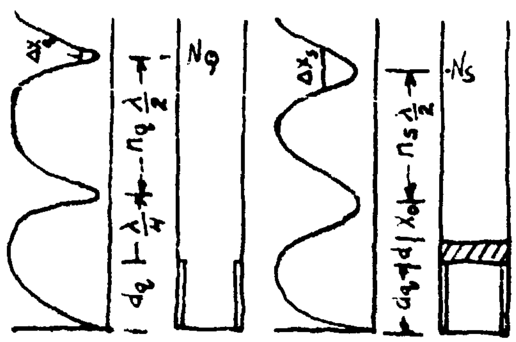
$$\frac{\tan \beta d}{\beta d} = \frac{\lambda}{2\pi d} \tan \frac{2\pi x_o}{\lambda}$$

βd is determined from tables tan x/x

$$\kappa' = \frac{u + \left(\frac{\lambda}{2\pi d} \beta d\right)^2}{1 + u}$$

$$\tan \delta = \frac{\Delta x \, \beta d \left(1 + \tan^2 \frac{2\pi x_o}{\lambda}\right)}{d\left[\beta d(1 + \tan^2 \beta d) - \tan \beta d\right]} - \tan \delta_w .$$

$$\tan \delta_w = \frac{\Delta x_a}{n_{a(\lambda/2)}} \cdot \frac{1}{1+u} .$$



General relations sample $\lambda/4$ from short:

$$x_o = N_s - N_q + n_{q(\lambda/2)} - n_{s(\lambda/2)} + \frac{\lambda}{4} - d,$$

$$\Delta x = \Delta x_s - (x_o + n_{s(\lambda/2)}) \frac{\Delta x_a}{n_{a(\lambda/2)}} ,$$

$$\frac{E_{min}}{E_{max}} = \frac{\pi \Delta x}{\lambda} - C_1,$$

$$\frac{Z_B}{Z_1} = \frac{\dfrac{E_{min}}{E_{max}} - j \tan \dfrac{2\pi x_o}{\lambda}}{1 - j \dfrac{E_{min}}{E_{max}} \tan \dfrac{2\pi x_o}{\lambda}}$$

For TE waves, nonmagnetic sample,

$$\frac{\coth \gamma_2 d}{\gamma_2 d} = \frac{1}{\gamma_1} \frac{Z_B}{Z_1}$$

γd is determined from charts or tables of $\coth x/x$

$$\kappa^* = \frac{u + \left(\frac{\lambda}{2\pi d} \cdot \gamma_2 d\right)^2}{1+u} .$$

Simplified calculations (for $\tan \delta < 0.1$ and $n\kappa'' < 0.2$, where n is number of wavelengths in sample)

$$\frac{\cot \beta d}{\beta d} = \frac{\lambda}{2\pi d} \tan \frac{2\pi x_o}{\lambda}$$

βd is determined from tables $\cot x/x$

$$\kappa' = \frac{u + \left(\frac{\lambda}{2\pi d} \cdot \beta d\right)^2}{1+u}$$

$$\tan \delta = \frac{\Delta x \beta d \left(1 + \cot \frac{2\pi x_o}{\lambda}\right)}{d\left[\beta d(1 + \cot^2 \beta d + \cot \beta d)\right]} - C_2 \tan \delta_w .$$

<u>Correction term for E_{min}/E_{max}</u>

The corrected value of E_{min}/E_{max} for values above 0.1 and less than 0.5 may be found by subtracting a correction term indicated in the following table from the values of $\pi\Delta x/\lambda$.

Table B-1. Subtraction term to obtain correct E_{min}/E_{max}

$\frac{\pi\Delta x}{\lambda}$	Substraction factor	$\frac{\pi\Delta x}{\lambda}$	Substraction factor	$\frac{\pi\Delta x}{\lambda}$	Substraction factor
0.02	0.0000	0.52	0.0750	1.02	0.3714
0.04	0.0000	0.54	0.0827	1.04	0.3869
0.06	0.00015	0.56	0.0909	1.06	0.4026
0.08	0.00034	0.58	0.100	1.08	0.4185
0.10	0.0007	0.60	0.108	1.10	0.4347
0.12	0.0012	0.62	0.118	1.12	0.4510
0.14	0.0018	0.64	0.127	1.14	0.4690
0.16	0.0027	0.66	0.137	1.16	0.4842
0.18	0.0038	0.68	0.148	1.18	0.5011
0.20	0.0052	0.70	0.159	1.20	0.5192
0.22	0.0068	0.72	0.170	1.22	0.5354
0.24	0.0088	0.74	0.181	1.24	0.5529
0.26	0.0110	0.76	0.193	1.26	0.5705
0.28	0.0136	0.78	0.205	1.28	0.5882
0.30	0.0166	0.80	0.217	1.30	0.6061
0.32	0.0199	0.82	0.230	1.32	0.6242
0.34	0.0236	0.84	0.243	1.34	0.6425
0.36	0.0277	0.86	0.256	1.36	0.6608
0.38	0.0322	0.88	0.270	1.38	0.6794
0.40	0.0371	0.90	0.293	1.40	0.6981
0.42	0.0424	0.92	0.298	1.42	0.7169
0.44	0.0481	0.94	0.312	1.44	0.7359
0.46	0.0542	0.96	0.326	1.46	0.7551
0.48	0.0608	0.98	0.341	1.48	0.7743
0.50	0.0677	0.100	0.356	1.50	0.7938

Appendix C

Tables of $\frac{\tan x}{x}$

The range of x is 0 to 26.9 radians in increments of 0.005 from 0 to 11.000 and in increments of 0.1 from 11.0 to 26.9. *

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
0.000	1.0000	0.200	1.0136	0.400	1.0570	0.600	1.1402	0.800	1.2870
.005	1.0000	.205	1.0142	.405	1.0585	.605	1.1430	.805	1.2919
.010	1.0000	.210	1.0150	.410	1.0601	.610	1.1458	.810	1.2969
.015	1.0001	.215	1.0157	.415	1.0617	.615	1.1486	.815	1.3019
.020	1.0001	.220	1.0165	.420	1.0633	.620	1.1515	.820	1.3070
0.025	1.0002	0.225	1.0172	0.425	1.0649	0.625	1.1544	0.825	1.3121
.030	1.0003	.230	1.0180	.430	1.0666	.630	1.1573	.830	1.3174
.035	1.0004	.235	1.0188	.435	1.0682	.635	1.1603	.835	1.3227
.040	1.0005	.240	1.0197	.440	1.0700	.640	1.1634	.840	1.3281
.045	1.0007	.245	1.0205	.445	1.0717	.645	1.1664	.845	1.3336
0.050	1.0008	0.250	1.0214	0.450	1.0735	0.650	1.1695	0.850	1.3392
.055	1.0010	.255	1.0223	.455	1.0752	.655	1.1727	.855	1.3449
.060	1.0012	.260	1.0232	.460	1.0771	.660	1.1759	.860	1.3506
.065	1.0014	.265	1.0241	.465	1.0789	.665	1.1792	.865	1.3565
.070	1.0016	.270	1.0250	.470	1.0808	.670	1.1825	.870	1.3624
0.075	1.0019	0.275	1.0260	0.475	1.0827	0.675	1.1858	0.875	1.3685
.080	1.0021	.280	1.0270	.480	1.0846	.680	1.1892	.880	1.3746
.085	1.0024	.285	1.0280	.485	1.0866	.685	1.1926	.885	1.3809
.090	1.0027	.290	1.0290	.490	1.0885	.690	1.1961	.890	1.3872
.095	1.0030	.295	1.0301	.495	1.0906	.695	1.1997	.895	1.3936
0.100	1.0033	0.300	1.0311	0.500	1.0926	0.700	1.2033	0.900	1.4002
.105	1.0037	.305	1.0322	.505	1.0947	.705	1.2069	.905	1.4068
.110	1.0041	.310	1.0333	.510	1.0968	.710	1.2106	.910	1.4136
.115	1.0044	.315	1.0344	.515	1.0989	.715	1.2144	.915	1.4205
.120	1.0048	.320	1.0355	.520	1.1011	.720	1.2182	.920	1.4275
0.125	1.0052	0.325	1.0368	0.525	1.1033	0.725	1.2220	0.925	1.4346
.130	1.0057	.330	1.0380	.530	1.1055	.730	1.2259	.930	1.4418
.135	[illegible]	.335	1.0392	.535	1.1078	.735	1.2299	.935	1.4492
.140	1.0066	.340	1.0404	.540	1.1101	.740	1.2339	.940	1.4566
.145	1.0071	.345	1.0417	.545	1.1124	.745	1.2380	.945	1.4642
0.150	1.0076	0.350	1.0429	0.550	1.1147	0.750	1.2421	0.950	1.4720
.155	1.0081	.355	1.0442	.555	1.1171	.755	1.2463	.955	1.4799
.160	1.0086	.360	1.0456	.560	1.1196	.760	1.2506	.960	1.4879
.165	1.0092	.365	1.0469	.565	1.1220	.765	1.2549	.965	1.4960
.170	1.0097	.370	1.0483	.570	1.1245	.770	1.2593	.970	1.5043
0.175	1.0103	0.375	1.0497	0.575	1.1270	0.775	1.2638	0.975	1.5128
.180	1.0109	.380	1.0511	.580	1.1296	.780	1.2683	.980	1.5214
.185	1.0116	.385	1.0525	.585	1.1322	.785	1.2729	.985	[illegible]
.190	1.0122	.390	1.0540	.590	1.1348	.790	1.2775	.990	1.5391
.195	1.0129	.395	1.0555	.595	1.1375	.795	1.2823	.995	1.5482

* Similar tables ("Tables of $\frac{\text{Tan } x}{x}$ for Radian Measure," T. W. Dakin and M. Rutter, Res. Rep. R-0040-7-A, Westinghouse Research Laboratories, East Pittsburgh, Pa.) were published in 1945. The intervals in x in these tables are: 0.001 from 0 to 0.1 radian, 0.001 from 0.1 to 3.15 radians, 0.01 from 3.15 to 6.3 radians, and 0.1 from 6.3 to 10 radians.

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
1.000	1.5574	1.200	2.1435	1.400	4.1413	1.600	-21.3953	1.800	-2.3813
.005	1.5668	.205	2.1666	.405	4.2535	.605	-18.2089	.805	-2.3221
.010	1.5764	.210	2.1904	.410	4.3726	.610	-15.8353	.810	-2.2655
.015	1.5862	.215	2.2148	.415	4.4994	.615	-13.9987	.815	-2.2111
.020	1.5962	.220	2.2400	.420	4.6346	.620	-12.5354	.820	-2.1590
1.025	1.6064	1.225	2.2659	1.425	4.7791	1.625	-11.3421	1.825	-2.1089
.030	1.6167	.230	2.2925	.430	4.9339	.630	-10.3504	.830	-2.0608
.035	1.6273	.235	2.3200	.435	5.1001	.635	- 9.5132	.835	-2.0144
.040	1.6381	.240	2.3483	.440	5.2790	.640	- 8.7970	.840	-1.9698
.045	1.6491	.245	2.3775	.445	5.4722	.645	- 8.1773	.845	-1.9269
1.050	1.6603	1.250	2.4077	1.450	5.6814	1.650	- 7.6359	1.850	-1.8854
.055	1.6717	.255	2.4387	.455	5.9087	.655	- 7.1588	.855	-1.8455
.060	1.6834	.260	2.4708	.460	6.1566	.660	- 6.7353	.860	-1.8069
.065	1.6953	.265	2.5040	.465	6.4279	.665	- 6.3567	.865	-1.7696
.070	1.7075	.270	2.5383	.470	6.7261	.670	- 6.0163	.870	-1.7336
1.075	1.7199	1.275	2.5737	1.475	7.0555	1.675	- 5.7086	1.875	-1.6988
.080	1.7326	.280	2.6104	.480	7.4212	.680	- 5.4290	.880	-1.6651
.085	1.7456	.285	2.6484	.485	7.8296	.685	- 5.2174	.885	-1.6325
.090	1.7588	.290	2.6878	.490	8.2885	.690	- 4.9404	.890	-1.6009
.095	1.7723	.295	2.7285	.495	8.8080	.695	- 4.7256	.895	-1.5703
1.100	1.7861	1.300	2.7708	1.500	9.4009	1.700	- 4.5274	1.900	-1.5406
.105	1.8003	.305	2.8148	.505	10.0840	.705	- 4.3440	.905	-1.5118
.110	1.8147	.310	2.8604	.510	10.8795	.710	- 4.1738	.910	-1.4838
.115	1.8295	.315	2.9078	.515	11.8176	.715	- 4.0155	.915	-1.4567
.120	1.8446	.320	2.9571	.520	12.9405	.720	- 3.8677	.920	-1.4304
1.125	1.8601	1.325	3.0084	1.525	14.3086	1.725	- 3.7295	1.925	-1.4048
.130	1.8759	.330	3.0619	.530	16.0120	.730	- 3.6001	.930	-1.3799
.135	1.8921	.335	3.1176	.535	18.1915	.735	- 3.4785	.935	-1.3557
.140	1.9087	.340	3.1758	.540	21.0787	.740	- 3.3641	.940	-1.3321
.145	1.9256	.345	3.2366	.545	25.0852	.745	- 3.2563	.945	-1.3092
1.150	1.9430	1.350	3.3002	1.550	31.0184	1.750	- 3.1545	1.950	-1.2869
.155	1.9609	.355	3.3667	.555	40.7078	.755	- 3.0583	.955	-1.2652
.160	1.9791	.360	3.4364	.560	59.3721	.760	- 2.9671	.960	-1.2440
.165	1.9979	.365	3.5094	.565	110.2371	.765	- 2.8806	.965	-1.2234
.170	2.0171	.370	3.5862	.570	799.8507	.770	- 2.7985	.970	-1.2033
1.175	2.0368	1.375	3.6668	1.575	-151.0386	1.775	- 2.7205	1.975	-1.1837
.180	2.0570	.380	3.7518	.580	- 68.7653	.780	- 2.6461	.980	-1.1646
.185	2.0778	.385	3.8413	.585	- 44.4161	.785	- 2.5753	.985	-1.1459
.190	2.0991	.390	3.9357	.590	- 32.7465	.790	- 2.5076	.990	-1.1277
.195	2.1210	.395	4.0356	.595	- 25.8984	.795	- 2.4430	.995	-1.1099

$\frac{\tan x}{x}$ *(continued)*

x	tan x / x	x	tan x / x	x	tan x / x	x	tan x / x	x	tan x / x
2.000	-1.0925	2.200	- .6245	2.400	- .3817	2.600	- .2314	2.800	- .1270
.005	-1.076	.205	- .6165	.405	- .3771	.605	- .2283	.805	- .1247
.010	-1.059	.210	- .6087	.410	- .3725	.610	- .2253	.810	- .1225
.015	-1.043	.215	- .6011	.415	- .3680	.615	- .2223	.815	- .1203
.020	-1.027	.220	- .5935	.420	- .3636	.620	- .2193	.820	- .1181
2.025	-1.011	2.225	- .5861	2.425	- .3592	2.625	- .2164	2.825	- .1160
.030	- .9963	.230	- .5787	.430	- .3549	.630	- .2135	.830	- .1138
.035	- .9814	.235	- .5715	.435	- .3506	.635	- .2106	.835	- .1117
.040	- .9669	.240	- .5644	.440	- .3463	.640	- .2077	.840	- .1098
.045	- .9527	.245	- .5574	.445	- .3421	.645	- .2049	.845	- .1074
2.050	- .9388	2.250	- .5505	2.450	- .3380	2.650	- .2021	2.850	- .1053
.055	- .9252	.255	- .5437	.455	- .3339	.655	- .1993	.855	- .1032
.060	- .9118	.260	- .5370	.460	- .3298	.660	- .1965	.860	- .1011
.065	- .8988	.265	- .5304	.465	- .3258	.665	- .1937	.865	- .09908
.070	- .8860	.270	- .5239	.470	- .3218	.670	- .1910	.870	- .09703
2.075	- .8734	2.275	- .5174	2.475	- .3179	2.675	- .1883	2.875	- .09499
.080	- .8611	.280	- .5111	.480	- .3140	.680	- .1856	.880	- .09296
.085	- .8490	.285	- .5048	.485	- .3101	.685	- .1829	.885	- .09094
.090	- .8372	.290	- .4987	.490	- .3063	.690	- .1803	.890	- .08894
.095	- .8256	.295	- .4926	.495	- .3025	.695	- .1777	.895	- .08695
2.100	- .8142	2.300	- .4866	2.500	- .2988	2.700	- .1751	2.900	- .08497
.105	- .8030	.305	- .4807	.505	- .2951	.705	- .1725	.905	- .08300
.110	- .7921	.310	- .4749	.510	- .2915	.710	- .1699	.910	- .08104
.115	- .7813	.315	- .4691	.515	- .2878	.715	- .1674	.915	- .07909
.120	- .7707	.320	- .4634	.520	- .2843	.720	- .1649	.920	- .07715
2.125	- .7604	2.325	- .4578	2.525	- .2807	2.725	- .1624	2.925	- .07523
.130	- .7502	.330	- .4523	.530	- .2772	.730	- .1599	.930	- .07331
.135	- .7401	.335	- .4468	.535	- .2737	.735	- .1574	.935	- .07141
.140	- .7303	.340	- .4414	.540	- .2703	.740	- .1550	.940	- .06951
.145	- .720[illegible]	.345	- .4361	.545	- .2669	.745	- .1526	.945	- .06763
2.150	- .7111	2.350	- .4309	2.550	- .2635	2.750	- .1502	2.950	- .06575
.155	- .7018	.355	- .4256	.555	- .2601	.755	- .1478	.955	- .06389
.160	- .6926	.360	- .4205	.560	- .2568	.760	- .1454	.960	- .06203
.165	- .6836	.365	- .4154	.565	- .2535	.765	- .1430	.965	- .06019
.170	- .6747	.370	- .4104	.570	- .2503	.770	- .1407	.970	- .05835
2.175	- .6660	2.375	- .4055	2.575	- .2471	2.775	- .1384	2.975	- .05652
.180	- .6574	.380	- .4006	.580	- .2439	.780	- .1361	.980	- .05470
.185	- .6490	.385	- .3958	.585	- .2407	.785	- .1338	.985	- .05289
.190	- .6407	.390	- .3910	.590	- .2376	.790	- .1315	.990	- .05109
.195	- .6325	.395	- .3863	.595	- .2345	.795	- .1292	.995	- .04930

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
3.000	- .04751	3.200	.01827	3.400	.07774	3.600	1371	3.800	.2036
.005	- .04574	.205	.01933	.405	.07920	.605	.1386	.805	.2054
.010	- .04397	.210	.02134	.410	.08066	.610	.1402	.810	.2073
.015	- .04221	.215	.02288	.415	.08212	.615	.1417	.815	.2091
.020	- .04046	.220	.02440	.420	.08358	.620	.1433	.820	.2110
3.025	- .03872	3.225	.02592	3.425	.08504	3.625	.1448	3.825	.2129
.030	- .03698	.230	.02744	.430	.08650	.630	.1464	.830	.2148
.035	- .03525	.235	.02896	.435	.08796	.635	.1479	.835	.2167
.040	- .03353	.240	.03047	.440	.08942	.640	.1495	.940	.2186
.045	- .03182	.245	.03198	.445	.09088	.645	.1511	.845	.2206
3.050	- .03011	3.250	.03349	3.450	.09234	3.650	.1527	3.850	.2225
.055	- .02841	.255	.03499	.455	.09380	.655	.1543	.855	.2245
.060	- .02672	.260	.03649	.460	.09527	.660	.1559	.860	.2265
.065	- .02504	.265	.03799	.465	.09673	.665	.1575	.865	.2285
.070	- .02336	.270	.03949	.470	.09820	.670	.1591	.870	.2305
3.075	- .02169	3.275	.04098	3.475	.09967	3.675	.1607	3.875	.2325
.080	- .02002	.280	.04247	.480	.1011	.680	.1623	.880	.2346
.085	- .01836	.285	.04395	.485	.1026	.685	.1639	.885	.2366
.090	- .01671	.290	.04544	.490	.1041	.690	.1656	.890	.2387
.095	- .01506	.295	.04693	.495	.1056	.695	.1672	.895	.2408
3.100	- .01342	3.300	.04841	3.500	.1070	3.700	.1688	3.900	.2429
.105	- .01179	.305	.04989	.505	.1085	.705	.1705	.905	.2451
.110	- .01016	.310	.05137	.510	.1100	.710	.1722	.910	.2472
.115	- .008538	.315	.05284	.515	.1115	.715	.1738	.915	.2494
.120	- .006921	.320	.05432	.520	.1129	.720	.1755	.920	.2516
3.125	- .005309	3.325	.05579	3.525	.1144	3.725	.1772	3.925	.2538
.130	- .003701	.330	.05726	.530	.1159	.730	.1789	.930	.2560
.135	- .002102	.335	.05873	.535	.1174	.735	.1806	.915	.2582
.140	- .000506	.340	.06020	.540	.1189	.740	.1823	.940	.2605
.145	.001086	.345	.06166	.545	.1204	.745	.1840	.945	.2628
3.150	.002670	3.350	.06313	3.550	.1219	3.750	.1857	3.950	.2651
.155	.004251	.355	.06459	.555	.1234	.755	.1875	.955	.2674
.160	.005827	.360	.06606	.560	.1249	.760	.1892	.960	.2698
.165	.007398	.365	.06752	.565	.1264	.765	.1910	.965	.2721
.170	.008969	.370	.06898	.570	.1279	.770	.1928	.970	.2745
3.175	.01052	3.375	.07044	3.575	.1294	3.775	.1945	3.975	.2770
.180	.01208	.380	.07190	.580	.1310	.780	.1963	.980	.2794
.185	.01364	.385	.07336	.585	.1325	.785	.1981	.985	.2819
.190	.01519	.390	.07482	.590	.1340	.790	.1999	.990	.2844
.195	.01673	.395	.07628	.595	.1355	.795	.2017	.995	.2869

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
4.000	.2895	4.200	.4233	4.400	.7037	4.600	1.926	4.800	-2.372
.005	.2920	.205	.4278	.405	.7151	.605	2.014	.805	-2.241
.010	.2946	.210	.4323	..410	.7269	.610	2.111	.810	-2.123
.015	.2973	.215	.4370	.415	.7391	.615	2.218	.815	-2.017
.020	.2999	.220	.4417	.420	.7516	.620	2.336	.820	-1.920
4.025	.3026	4.225	.4465	4.425	.7646	4.625	2.468	4.825	-1.833
.030	.3054	.230	.4515	.430	.7780	.630	2.616	.830	-1.752
.035	.3081	.235	.4565	.435	.7919	.635	2.782	.835	-1.678
.040	.3109	.240	.4616	.440	.8063	.640	2.972	.840	-1.610
.045	.3137	.245	.4668	.445	.8212	.645	3.190	.845	-1.547
4.050	.3166	4.250	.4721	4.450	.8367	4.650	3.443	4.850	-1.489
.055	.3195	.255	.4775	.455	.8528	.655	3.739	.855	-1.434
.060	.3224	.260	.4830	.460	.8694	.660	4.093	.860	-1.384
.065	.3254	.265	.4886	.465	.8868	.665	4.520	.865	-1.336
.070	.3284	.270	.4944	.470	.9048	.670	5.049	.870	-1.292
4.075	.3314	4.275	.5003	4.475	.9236	4.675	5.719	4.875	-1.250
.080	.3345	.280	.5063	.480	.9432	.680	6.595	.880	-1.211
.085	.3376	.285	.5124	.485	.9636	.685	7.791	.885	-1.174
.090	.3408	.290	.5186	.490	.9849	.690	9.522	.890	-1.139
.095	.3440	.295	.5251	.495	1.007	.695	12.25	.895	-1.106
4.100	.3472	4.300	.5316	4.500	1.031	4.700	17.17	4.900	-1.075
.105	.3505	.305	.5383	.505	1.055	.705	28.76	.905	-1.045
.110	.3538	.310	.5451	.510	1.081	.710	88.87	.910	-1.017
.115	.3572	.315	.5522	.515	1.110	.715	-81.23	.915	- .9904
.120	.3606	.320	.5593	.520	1.136	.720	-27.84	.920	- .9649
4.125	.3641	4.325	.5667	4.525	1.166	4.725	-16.78	4.925	- .9406
.130	.3677	.330	.5742	.530	1.197	.730	-12.00	.930	- .9173
.135	.3712	.335	.5820	.535	1.230	.735	- 9.337	.935	- .8952
.140	.3749	.340	.5899	.540	1.265	.740	- 7.637	.940	- .8739
.145	.3787	.345	.5980	.545	1.302	.745	- 6.459	.945	- .8536
4.150	.3823	4.350	.6063	4.550	1.342	4.750	- 5.594	4.950	- .8341
155	.3861	.355	.6149	.555	1.383	.755	- 4.932	.955	- .8154
.160	.3900	.360	.6237	.560	1.428	.760	- 4.409	.960	- .7975
.165	.3939	.365	.6327	.565	1.476	.765	- 3.985	.965	- .7803
.170	.3979	.370	.6420	.570	1.526	.770	- 3.635	.970	- .7637
4.175	.4020	4.375	.6516	4.575	1.581	4.775	- 3.340	4.975	- .7477
.180	.4061	.380	.6614	.580	1.640	.780	- 3.089	.980	- .7323
.185	.4103	.385	.6715	.585	1.703	.785	- 2.873	.985	- .7175
.190	.4145	.390	.6819	.590	1.771	.790	- 2.684	.990	- .7032
.195	.4189	.395	.6927	.595	1.845	.795	- 2.519	.995	- .6894

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
5.000	- .6761	5.200	- .3626	5.400	- .2255	5.600	- .1453	5.800	- .09046
.005	- .6632	.205	- .3579	.405	- .2230	.605	- .1437	.805	- .08929
.010	- .6507	.210	- .3533	.410	- .2205	.610	- .1421	.810	- .08812
.015	- .6387	.215	- .3488	.415	- .2181	.615	- .1406	.815	- .08696
.020	- .6270	.220	- .3444	.420	- .2157	.620	- .1390	.820	- .08581
5.025	- .6157	5.225	- .3401	5.425	- .2133	5.625	- .1375	5.825	- .08467
.030	- .6047	.230	- .3358	.430	- .2110	.630	- .1359	.830	- .08353
.035	- .5941	.235	- .3316	.435	- .2087	.635	- .1344	.835	- .08240
.040	- .5838	.240	- .3275	.440	- .2064	.640	- .1329	.840	- .08128
.045	- .5738	.245	- .3235	.445	- .2041	.645	- .1314	.845	- .08016
5.050	- .5641	5.250	- .3195	5.450	- .2019	5.650	- .1299	5.850	- .07906
.055	- .5546	.255	- .3156	.455	- .2001	.655	- .1284	.855	- .07795
.060	- .5454	.260	- .3117	.460	- .1975	.660	- .1270	.860	- .07686
.065	- .5365	.265	- .3080	.465	- .1954	.665	- .1255	.865	- .07577
.070	- .5278	.270	- .3043	.470	- .1933	.670	- .1241	.870	- .07469
5.075	- .5194	5.275	- .3006	5.475	- .1912	5.675	- .1227	5.875	- .07361
.080	- .5111	.280	- .2970	.480	- .1891	.680	- .1213	.880	- .07254
.085	- .5031	.285	- .2935	.485	- .1870	.685	- .1199	.885	- .07148
.090	- .4953	.290	- .2900	.490	- .1850	.690	- .1185	.890	- .07042
.095	- .4877	.295	- .2866	.495	- .1830	.695	- .1171	.895	- .06937
5.100	- .4803	5.300	- .2833	5.500	- .1810	5.700	- .1157	5.900	- .06832
.105	- .4730	.305	- .2799	.505	- .1790	.705	- .1144	.905	- .06728
.110	- .4660	.310	- .2767	.510	- .1771	.710	- .1130	.910	- .06625
.115	- .4591	.315	- .2735	.515	- .1752	.715	- .1117	.915	- .06522
.120	- .4523	.320	- .2703	.520	- .1733	.720	- .1104	.920	- .06420
5.125	- .4457	5.325	- .2672	5.525	- .1714	5.725	- .1091	5.925	- .06318
.130	- .4393	.330	- .2641	.530	- .1695	.730	- .1078	.930	- .06216
.135	- .4330	.335	- .2611	.535	- .1677	.735	- .1065	.935	- .06116
.140	- .4269	.340	- .2581	.540	- .1659	.740	- .1052	.940	- .06015
.145	- .4209	.345	- .2552	.545	- .1641	.745	- .1039	.945	- .05916
5.150	- .4150	5.350	- .2523	5.550	- .1623	5.750	- .1026	5.950	- .05817
.155	- .4093	.355	- .2494	.555	- .1605	.755	- .1014	.955	- .05718
.160	- .4036	.360	- .2466	.560	- .1588	.760	- .1001	.960	- .05620
.165	- .3981	.365	- .2439	.565	- .1570	.765	- .09890	.965	- .05522
.170	- .3928	.370	- .2411	.570	- .1553	.770	- .09767	.970	- .05424
5.175	- .3875	5.375	- .2384	5.575	- .1536	5.775	- .09645	5.975	- .05328
.180	- .3823	.380	- .2358	.580	- .1519	.780	- .09523	.980	- .05231
.185	- .3772	.385	- .2331	.585	- .1503	.785	- .09403	.985	- .05135
.190	- .3723	.390	- .2305	.590	- .1486	.790	- .09283	.990	- .05040
.195	- .3674	.395	- .2280	.595	- .1470	.795	- .09164	.995	- .04945

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
6.000	- .04850	6.200	- .01345	6.400	.01834	6.600	.04968	6.800	.08358
.005	- .04756	.205	- .01263	.405	.01911	.605	.05048	.805	.08449
.010	- .04662	.210	- .01181	.410	.01989	.610	.05128	.810	.08541
.015	- .04569	.215	- .01099	.415	.02067	.615	.05209	.815	.08633
.020	- .04476	.220	- .01017	.420	.02145	.620	.05289	.820	.08726
6.025	- .04383	6.225	- .009357	6.425	.02222	6.625	.05370	6.825	.08819
.030	- .04291	.230	- .008544	.430	.02299	.630	.05451	.830	.08913
.035	- .04199	.235	- .007733	.435	.02378	.635	.05533	.835	.09007
.040	- .04107	.240	- .006924	.440	.02455	.640	.05614	.840	.09101
.045	- .04016	.245	- .006117	.445	.02533	.645	.05696	.845	.09196
6.050	- .03926	6.250	- .005311	6.450	.02611	6.650	.05778	6.850	.09292
.055	- .03835	.255	- .004506	.455	.02688	.655	.05860	.855	.09388
.060	- .03745	.260	- .003704	.460	.02766	.660	.05942	.860	.09484
.065	- .03656	.265	- .002902	.465	.02844	.665	.06024	.865	.09582
.070	- .03566	.270	- .002102	.470	.02922	.670	.06107	.870	.09679
6.075	- .03477	6.275	- .001304	6.475	.02999	6.675	.06190	6.875	.09777
.080	- .03389	.280	- .0005064	.480	.03077	.680	.06273	.880	.09876
.085	- .03300	.285	.0002896	.485	.03155	.685	.06357	.885	.09975
.090	- .03212	.290	.001084	.490	.03233	.690	.06440	.890	.1008
.095	- .03124	.295	.001878	.495	.03311	.695	.06524	.895	.1018
6.100	- .03037	6.300	.002670	6.500	.03389	6.700	.06608	6.900	.1028
.105	- .02950	.305	.003461	.505	.03467	.705	.06693	.905	.1038
.110	- .02863	.310	.004251	.510	.03545	.710	.06778	.910	.1048
.115	- .02777	.315	.005040	.515	.03623	.715	.06863	.915	.1058
.120	- .02690	.320	.005829	.520	.03702	.720	.06948	.920	.1069
6.125	- .02604	6.325	.006616	6.525	.03780	6.725	.07034	6.925	.1079
.130	- .02518	.330	.007402	.530	.03858	.730	.07119	.930	.1090
.135	- .02433	.335	.008187	.535	.03937	.735	.07206	.935	.1100
.140	- .02348	.340	.008972	.540	.04016	.740	.07292	.940	.1111
.145	- [illegible]	.345	.009756	.545	.04094	.745	.07379	.945	.1122
6.150	- .02178	6.350	.01054	6.550	.04173	6.750	.07466	6.950	.1133
.155	- .02094	.355	.01132	.555	.04252	.755	.07554	.955	.1143
.160	- .02010	.360	.01210	.560	.04331	.760	.07642	.960	.1154
.165	- .01926	.365	.01288	.565	.04410	.765	.07730	.965	.1165
.170	- .01842	.370	.01366	.570	.04489	.770	.07818	.970	.1176
6.175	- .01759	6.375	.01444	6.575	.04569	6.775	.07907	6.975	.1188
.180	- .01676	.380	.01522	.580	.04648	.780	.07997	.980	.1199
.185	- .01593	.385	.01600	.585	.04728	.785	.08086	.985	.1210
.190	- .01510	.390	.01678	.590	.04808	.790	.08177	.990	.1222
.195	- .01434	.395	.01756	.595	.04888	.795	.08267	.995	.1233

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
7.000	.1245	7.200	.1812	7.400	.2769	7.600	.5069	7.800	2.373
.005	.1257	.205	.1830	.405	.2803	.605	.5172	.805	2.614
.010	.1268	.210	.1847	.410	.2837	.610	.5279	.810	2.910
.015	.1280	.215	.1866	.415	.2872	.615	.5390	.815	3.281
.020	.1292	.220	.1884	.420	.2908	.620	.5506	.820	3.762
7.025	.1305	7.225	.1902	7.425	.2945	7.625	.5627	7.825	4.410
.030	.1317	.230	.1921	.430	.2982	.630	.5753	.830	5.324
.035	.1329	.235	.1916	.435	.3020	.635	.5885	.835	6.724
.040	.1342	.240	.1960	.440	.3059	.640	.6023	.840	9.123
.045	.1354	.245	.1979	.445	.3099	.645	.6168	.845	14.19
7.050	.1367	7.250	.1999	7.450	.3140	7.650	.6319	7.850	31.99
.055	.1379	.255	.2019	.455	.3182	.655	.6478	.855	-125.0
.060	.1392	.260	.2040	.460	.3225	.660	.6645	.860	- 21.14
.065	.1405	.265	.2060	.465	.3268	.665	.6821	.865	- 11.54
.070	.1418	.270	.2081	.470	.3313	.670	.7007	.870	- 7.932
7.075	.1432	7.275	.2103	7.475	.3359	7.675	.7202	7.875	- 6.041
.080	.1445	.280	.2124	.480	.3407	.680	.7409	.880	- 4.876
.085	.1459	.285	.2146	.485	.3455	.685	.7627	.885	- 4.087
.090	.1472	.290	.2169	.490	.3505	.690	.7859	.890	- 3.517
.095	.1486	.295	.2191	.495	.3556	.695	.8105	.895	- 3.086
7.100	.1500	7.300	.2214	7.500	.3608	7.700	.8368	7.900	- 2.749
.105	.1514	.305	.2238	.505	.3662	.705	.8647	.905	- 2.478
.110	.1528	.310	.2262	.510	.3717	.710	.8946	.910	- 2.255
.115	.1542	.315	.2286	.515	.3774	.715	.9266	.915	- 2.068
.120	.1557	.320	.2310	.520	.3833	.720	.9610	.920	- 1.910
7.125	.1572	7.325	.2335	7.525	.3893	7.725	.9981	7.925	- 1.774
.130	.1586	.330	.2361	.530	.3955	.730	1.038	.930	- 1.656
.135	.1601	.335	.2387	.535	.4019	.735	1.082	.935	- 1.552
.140	.1616	.340	.2413	.540	.4084	.740	1.129	.940	- 1.461
.145	.1632	.345	.2440	.545	.4152	.745	1.180	.945	- 1.379
7.150	.1647	7.350	.2467	7.550	.4222	7.750	1.237	7.950	- 1.306
.155	.1663	.355	.2495	.555	.4294	.755	1.300	.955	- 1.240
.160	.1679	.360	.2523	.560	.4369	.760	1.367	.960	- 1.181
.165	.1695	.365	.2552	.565	.4446	.765	1.44[illegible]	.965	- 1.126
.170	.1711	.370	.2581	.570	.4526	.770	1.52[illegible]	.970	- 1.077
7.175	.1727	7.375	.2611	7.575	.4609	7.775	1.625	7.975	- 1.031
.180	.1744	.380	.2641	.580	.4694	.780	1.734	.980	- .9892
.185	.1760	.385	.2672	.585	.4783	.785	1.859	.985	- .9504
.190	.1777	.390	.2704	.590	.4875	.790	2.004	.990	- .9145
.195	.1795	.395	.2736	.595	.4970	.795	2.173	.995	- .8811

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
8.000	- .8500	8.200	- .3383	8.400	- .1959	8.600	- .1258	8.800	- .08195
.005	- .8209	.205	- .3328	.405	- .1936	.605	- .1245	.805	- .08104
.010	- .7937	.210	- .3275	.410	- .1913	.610	- .1232	.810	- .08014
.015	- .7682	.215	- .3224	.415	- .1891	.615	- .1219	.815	- .07925
.020	- .7441	.220	- .3174	.420	- .1869	.620	- .1206	.820	- .07836
8.025	- .7215	8.225	- .3125	8.425	- .1848	8.625	- .1193	8.825	- .07749
.030	- .7002	.230	- .3078	.430	- .1826	.630	- .1181	.830	- .07661
.035	- .6801	.235	- .3031	.435	- .1806	.635	- .1168	.835	- .07575
.040	- .6609	.240	- .2986	.440	- .1785	.640	- .1156	.840	- .07489
.045	- .6428	.245	- .2942	.445	- .1765	.645	- .1144	.845	- .07404
8.050	- .6256	8.250	- .2899	8.450	- .1745	8.650	- .1132	8.850	- .07319
.055	- .6093	.255	- .2857	.455	- .1725	.655	- .1120	.855	- .07235
.060	- .5937	.260	- .2816	.460	- .1706	.660	- .1108	.860	- .07152
.065	- .5788	.265	- .2776	.465	- .1687	.665	- .1096	.865	- .07069
.070	- .5647	.270	- .2737	.470	- .1668	.670	- .1085	.870	- .06986
8.075	- .5512	8.275	- .2699	8.475	- .1649	8.675	- .1073	8.875	- .06904
.080	- .5382	.280	- .2661	.480	- .1631	.680	- .1062	.880	- .06824
.085	- .5258	.285	- .2625	.485	- .1613	.685	- .1051	.885	- .06743
.090	- .5140	.290	- .2589	.490	- .1595	.690	- .1040	.890	- .06663
.095	- .5026	.295	- .2554	.495	- .1578	.695	- .1029	.895	- .06584
8.100	- .4917	8.300	- .2520	8.500	- .1560	8.700	- .1018	8.900	- .06505
.105	- .4812	.305	- .2486	.505	- .1543	.705	- .1007	.905	- .06426
.110	- .4711	.310	- .2453	.510	- .1527	.710	- .0996	.910	- .06348
.115	- .4613	.315	- .2421	.515	- .1510	.715	- .09858	.915	- .06271
.120	- .4520	.320	- .2390	.520	- .1494	.720	- .09753	.920	- .06194
8.125	- .4430	8.325	- .2359	8.525	- .1478	8.725	- .09649	8.925	- .06118
.130	- .4343	.330	- .2329	.530	- .1462	.730	- .09546	.930	- .06042
.135	- .4259	.335	- .2299	.535	- .1446	.735	- .09444	.935	- .05966
.140	- .4177	.340	- .2270	.540	- .1430	.740	- .09343	.940	- .05892
.145	- .4099	.345	- .2241	.545	- .1415	.745	- .09243	.945	- .05817
8.150	- .4023	8.350	- .2213	8.550	- .1400	8.750	- .09144	8.950	- .05743
.155	- .3950	.355	- .2186	.555	- .1385	.755	- .09045	.955	- .05669
.160	- .3879	.360	- .2159	.560	- .1370	.760	- .08947	.960	- .05596
.165	- .3810	.365	- .2132	.565	- .1355	.765	- .08851	.965	- .05523
.170	- .3743	.370	- .2106	.570	- .1341	.770	- .08755	.970	- .05451
8.175	- .3679	8.375	- .2081	8.575	- .1327	8.775	- .08659	8.975	- .05379
.180	- .3616	.380	- .2055	.580	- .1313	.780	- .08565	.980	- .05308
.185	- .3555	.385	- .2031	.585	- .1299	.785	- .08471	.985	- .05237
.190	- .3496	.390	- .2006	.590	- .1285	.790	- .08378	.990	- .05166
.195	- .3438	.395	- .1982	.595	- .1272	.795	- .08286	.995	- .05096

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
9.000	-.05026	9.200	-.02485	9.400	-.002637	9.600	.01844	9.800	.04019
.005	-.04956	.205	-.02427	.405	-.002103	.605	.01897	.805	.04076
.010	-.04887	.210	-.02369	.410	-.001571	.610	.01950	.810	.04133
.015	-.04818	.215	-.02310	.415	-.001039	.615	.02003	.815	.04191
.020	-.04750	.220	-.02253	.420	-.0005074	.620	.02055	.820	.04248
9.025	-.04682	9.225	-.02195	9.425	.00002334	9.625	.02108	9.825	.04306
.030	-.04614	.230	-.02137	.430	.0005536	.630	.02161	.830	.04364
.035	-.04547	.235	-.02080	.435	.001083	.635	.02215	.835	.04422
.040	-.04480	.240	-.02023	.440	.001612	.640	.02268	.840	.04480
.045	-.04413	.245	-.01966	.445	.002141	.645	.02321	.845	.04539
9.050	-.04347	9.250	-.01909	9.450	.002669	9.650	.02374	9.850	.04597
.055	-.04281	.255	-.01852	.455	.003197	.655	.02428	.855	.04656
.060	-.04215	.260	-.01796	.460	.003725	.660	.02481	.860	.04716
.065	-.04149	.265	-.01739	.465	.004252	.665	.02534	.865	.04775
.070	-.04084	.270	-.01683	.470	.004778	.670	.02588	.870	.04835
9.075	-.04020	9.275	-.01627	9.475	.005305	9.675	.02642	9.875	.04894
.080	-.03955	.280	-.01571	.480	.005831	.680	.02695	.880	.04955
.085	-.03891	.285	-.01515	.485	.006357	.685	.02749	.885	.05015
.090	-.03827	.290	-.01460	.490	.006882	.690	.02803	.890	.05076
.095	-.03763	.295	-.01404	.495	.007408	.695	.02857	.895	.05136
9.100	-.03700	9.300	-.01349	9.500	.007933	9.700	.02911	9.900	.05197
.105	-.03637	.305	-.01293	.505	.008458	.705	.02965	.905	.05259
.110	-.03574	.310	-.01238	.510	.008983	.710	.03020	.910	.05321
.115	-.03512	.315	-.01183	.515	.009508	.715	.03074	.915	.05382
.120	-.03449	.320	-.01128	.520	.01003	.720	.03129	.920	.05445
9.125	-.03387	9.325	-.01074	9.525	.01056	9.725	.03183	9.925	.05507
.130	-.03326	.330	-.01019	.530	.01108	.730	.03238	.930	.05570
.135	-.03264	.335	-.009643	.535	.01161	.735	.03293	.935	.05633
.140	-.03203	.340	-.009099	.540	.01213	.740	.03348	.940	.05696
.145	-.03142	.345	-.008555	.545	.01266	.745	.03403	.945	.05760
9.150	-.03081	9.350	-.008013	9.550	.01318	9.750	.03458	9.950	.05824
.155	-.03020	.355	-.007471	.555	.01371	.755	.03514	.955	.05889
.160	-.02960	.360	-.006931	.560	.01423	.760	.03569	.960	.05953
.165	-.02900	.365	-.006391	.565	.01476	.765	.03625	.965	.06018
.170	-.02840	.370	-.005852	.570	.01528	.770	.03681	.970	.06084
9.175	-.02780	9.375	-.005314	9.575	.01581	9.775	.03737	9.975	.06149
.180	-.02721	.380	-.004777	.580	.01633	.780	.03793	.980	.06216
.185	-.02662	.385	-.004241	.585	.01686	.785	.03849	.985	.06282
.190	-.02603	.390	-.003705	.590	.01739	.790	.03906	.990	.06349
.195	-.02544	.395	-.003171	.595	.01791	.795	.03962	.995	.06416

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
10.000	.06484	10.200	[illegible]	10.400	.1419	10.600	.2259	10.800	.4674
.005	.06552	.205	.09698	.405	.1434	.605	.2290	.805	.4797
.010	.06620	.210	.09791	.410	.1449	.610	.2322	.810	.4927
.015	.06689	.215	.09884	.415	.1464	.615	.2355	.815	.5066
.020	.06759	.220	.09979	.420	.1479	.620	.2388	.820	[illegible]
10.025	.06828	10.225	.1007	10.425	.1495	10.625	.2422	10.825	.5363
.030	.06898	.230	.1017	.430	.1510	.630	.2458	.830	.5525
.035	.06968	.235	.1027	.435	.1527	.635	.2494	.835	.5698
.040	.07039	.240	.1037	.440	.1543	.640	.2531	.840	.5882
.045	.07110	.245	.1047	.445	.1560	.645	.2569	.845	.6077
10.050	.07182	10.250	.1057	10.450	.1576	10.650	.2608	10.850	.6286
.055	.07255	.255	.1067	.455	.1594	.655	.2648	.855	.6510
.060	.07327	.260	.1077	.460	.1611	.660	.2690	.860	.6750
.065	.07401	.265	.1087	.465	.1629	.665	.2732	.865	.7009
.070	.07475	.270	.1098	.470	.1647	.670	.2776	.870	.7287
10.075	.07549	10.275	.1108	10.475	.1665	10.675	.2821	10.875	.7589
.080	.07624	.280	.1119	.480	.1684	.680	.2868	.880	.7917
.085	.07699	.285	.1130	.485	.1703	.685	.2916	.885	.8274
.090	.07775	.290	.1141	.490	.1722	.690	.2965	.890	.8665
.095	.07852	.295	.1152	.495	.1742	.695	.3016	.895	.9095
10.100	.07929	10.300	.1163	10.500	.1762	10.700	.3069	10.900	[illegible]
.105	.08006	.305	.1174	.505	.1782	.705	.3124	.905	1.010
.110	.08084	.310	.1186	.510	.1803	.710	.3180	.910	1.069
.115	.08163	.315	.1197	.515	.1824	.715	.3239	.915	1.135
.120	.08243	.320	.1209	.520	.1846	.720	.3299	.920	1.209
10.125	.08323	10.325	.1221	10.525	.1868	10.725	.3361	10.925	1.295
.130	.08403	.330	.1233	.530	.1890	.730	.3426	.930	1.391
.135	[illegible]	.335	.1245	.535	.1913	.735	.3494	.935	1.508
.140	[illegible]	.340	.1258	.540	.1936	.740	.3563	.940	1.643
.145	.08649	.345	.1270	.545	.1960	.745	.3636	.945	1.805
10.150	.08732	10.350	.1283	10.550	.1985	10.750	.3712	10.950	2.007
.155	.08816	.355	.1295	.555	.2009	.755	.3790	.955	2.248
.160	.08901	.360	.1308	.560	.2035	.760	.3872	.960	2.564
.165	.08987	.365	.1322	.565	.2061	.765	.3957	.965	2.962
.170	.09073	.370	.1335	.570	.2087	.770	.4046	.970	3.564
10.175	.09160	10.375	.1348	10.575	.2114	10.775	.4139	10.975	4.428
.180	.09247	.380	.1362	.580	.2142	.780	.4236	.980	5.848
.185	.09336	.385	.1376	.585	.2170	.785	.4338	.985	8.600
.190	.09425	.390	.1390	.590	.2199	.790	.4445	.990	[illegible]
.195	.09515	.395	.1405	.595	.2229	.795	.4556	.995	[illegible]

$\frac{\tan x}{x}$ *(continued)*

x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$	x	$\frac{\tan x}{x}$
11.0	-20.66	15.0	- .05707	19.0	.00797	23.0	.06775
.1	- .8598	.1	- .04609	.1	.01339	.1	.08696
.2	- .4307	.2	- .03663	.2	.01904	.2	.1139
.3	- .2817	.3	- .02825	.3	.02506	.3	.1601
.4	- .2050	.4	- .02066	.4	.03163	.4	.2617
11.5	- .1575	15.5	- .01361	19.5	.03902	23.5	.6864
.6	- .1248	.6	- .00700	.6	.04757	.6	- 1.142
.7	- .1006	.7	- .00051	.7	.05783	.7	- .3035
.8	- .08153	.8	.005842	.8	.07069	.8	- .1732
.9	- .06608	.9	.01223	.9	.08768	.9	- .1190
12.0	- .05298	16.0	.01879	20.0	.1118	24.0	- .08894
.1	04160	.1	.02568	.1	.1500	.1	- .06952
.2	.03145	.2	.03309	.2	.2209	.2	- .05572
.3	- .02218	.3	.04126	.3	.4072	.3	- .04524
.4	- .01355	.4	.05053	.4	2.403	.4	- .03688
12.5	- .00532	16.5	.06141	20.5	- .6112	24.5	- .02993
.6	.00267	.6	.07468	.6	- .2672	.6	- .02397
.7	.01058	.7	.09163	.7	- .1682	.7	- .01870
.8	.01859	.8	.1147	.8	- .1205	.8	- .01394
.9	.02687	.9	.1487	.9	- .09198	.9	- .00952
13.0	.03562	17.0	.2055	21.0	- .07273	25.0	- .00534
.1	.04510	.1	.3236	.1	- .05865	.1	- .00130
.2	.05566	.2	.7360	.2	- .04771	.2	.00267
.3	.06777	.3	-2.726	.3	- .03884	.3	.00668
.4	.08219	.4	- .4719	.4	- .03136	.4	.01078
13.5	.1001	17.5	- .2541	21.5	- .02487	25.5	.01509
.6	.1234	.6	- .1708	.6	- .01909	.6	.01971
.7	.1562	.7	- .1261	.7	- .01381	.7	.02479
.8	.2067	.8	- .09785	.8	- .008876	.8	.03053
.9	.2970	.9	- .07806	.9	- .00417	.9	.03724
14.0	.5174	18.0	- .06319	22.0	.00004	26.0	.04534
.1	1.906	.1	- .05143	.1	.00495	.1	.05559
.2	-1.120	.2	- .04174	.2	.00955	.2	.06929
.3	- .4258	.3	- .03347	.3	.01431	.3	.08906
.4	- .2581	.4	- .02623	.4	.01934	.4	.1209
14.5	- .1817	18.5	- .01971	22.5	.02480	26.5	.1829
.6	- .1373	.6	- .01370	.6	.03085	.6	.3620
.7	- .1078	.7	- .00806	.7	.03778	.7	10.70
.8	- .08655	.8	- .00264	.8	.04597	.8	- .3855
.9	- .07021	.9	.00267	.9	.05604	.9	- .1868

Appendix D

Tables of $\frac{\cot x}{x}$

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
0.100	99.67	0.150	44.11	0.200	24.67	0.250	15.66	0.300	10.78
.101	97.70	.151	43.52	.201	24.42	.251	15.54	.301	10.70
.102	95.78	.152	42.95	.202	24.17	.252	15.41	.302	10.63
.103	93.93	.153	42.38	.203	23.93	.253	15.29	.303	10.56
.104	92.12	.154	41.83	.204	23.70	.254	15.16	.304	10.48
0.105	90.37	0.155	41.29	0.205	23.46	0.255	15.04	0.305	10.41
.106	88.67	.156	40.26	.206	23.23	.256	14.92	.306	10.34
.107	87.01	.157	40.24	.207	23.00	.257	14.80	.307	10.27
.108	85.40	.158	39.72	.208	22.78	.258	14.69	.308	10.20
.109	83.83	.159	39.22	.209	22.56	.259	14.57	.309	10.14
0.110	82.31	0.160	38.73	0.210	22.34	0.260	14.46	0.310	10.07
.111	80.83	.161	38.24	.211	22.13	.261	14.34	.311	10.00
.112	79.38	.162	37.77	.212	21.92	.262	14.23	.312	9.937
.113	77.98	.163	37.30	.213	21.71	.263	14.12	.313	9.872
.114	76.61	.164	36.85	.214	21.50	.264	14.01	.314	9.807
0.115	75.28	0.165	36.40	0.215	21.30	0.265	13.91	0.315	9.743
.116	73.98	.166	35.96	.216	21.10	.266	13.80	.316	9.679
.117	72.72	.167	35.52	.217	20.90	.267	13.69	.317	9.616
.118	71.48	.168	35.10	.218	20.72	.268	13.59	.318	9.553
.119	70.28	.169	34.68	.219	20.52	.269	13.48	.319	9.491
0.120	69.11	0.170	34.27	0.220	20.33	0.270	13.38	0.320	9.430
.121	67.97	.171	33.86	.221	20.14	.271	13.28	.321	9.369
.122	66.85	.172	33.47	.222	19.96	.272	13.18	.322	9.309
.123	65.76	.173	33.08	.223	19.77	.273	13.08	.323	9.249
.124	64.70	.174	32.70	.224	19.60	.274	12.98	.324	9.190
0.125	63.66	0.175	32.32	0.225	19.42	0.275	12.89	0.325	9.132
.126	62.65	.176	31.95	.226	19.24	.276	12.79	.326	9.074
.127	61.67	.177	31.58	.227	19.07	.277	12.70	.327	9.016
.128	60.70	.178	31.23	.228	18.90	.278	12.60	.328	8.959
.129	59.76	.179	30.88	.229	18.73	.279	12.51	.329	8.903
0.130	58.84	0.180	30.53	0.230	18.57	0.280	12.42	0.330	8.847
.131	57.94	.181	30.19	.231	18.40	.281	12.33	.331	8.792
.132	57.06	.182	29.86	.232	18.24	.282	12.24	.332	8.736
.133	56.20	.183	29.53	.233	18.08	.283	12.15	.333	8.682
.134	55.36	.184	29.20	.234	17.93	.284	12.06	.334	8.628
0.135	54.53	0.185	28.89	0.235	17.77	0.285	11.98	0.335	8.574
.136	53.73	.186	28.57	.236	17.62	.286	11.89	.336	8.522
.137	52.94	.187	28.26	.237	17.47	.287	11.80	.337	8.469
.138	52.18	.188	27.96	.238	17.32	.288	11.72	.338	8.417
.139	51.42	.189	27.66	.239	17.17	.289	11.68	.339	8.366
0.140	50.69	0.190	27.37	0.240	17.03	0.290	11.56	0.340	8.315
.141	49.96	.191	27.08	.241	16.88	.291	11.47	.341	8.264
.142	49.26	.192	26.79	.242	16.74	.292	11.39	.342	8.214
.143	48.57	.193	26.51	.243	16.60	.293	11.31	.343	8.164
.144	47.89	.194	26.24	.244	16.46	.294	11.23	.344	8.114
0.145	47.23	0.195	25.96	0.245	16.33	0.295	11.16	0.345	8.066
.146	46.58	.196	25.70	.246	16.19	.296	11.08	.346	8.017
.147	45.94	.197	25.43	.247	16.06	.297	11.00	.347	7.969
.148	45.32	.198	25.17	.248	15.92	.298	10.92	.348	7.921
.149	44.71	.199	24.92	.249	15.79	.299	10.85	.349	7.874
0.150	44.11	0.200	24.67	0.250	15.66	0.300	10.78	0.350	7.827

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
0.350	7.827	0.400	5.913	0.450	4.600	0.500	3.661	0.550	2.965
.351	7.781	.401	5.882	.451	4.578	.501	3.645	.551	2.954
.352	7.735	.402	5.851	.452	4.567	.502	3.629	.552	2.941
.353	7.689	.403	5.820	.453	4.535	.503	3.613	.553	2.930
.354	7.644	.404	5.790	.454	4.514	.504	3.598	.554	2.918
0.355	7.599	0.405	5.767	0.455	4.492	0.505	3.582	0.555	2.906
.356	7.554	.406	5.730	.456	4.471	.506	3.567	.556	2.894
.357	7.510	.407	5.700	.457	4.450	.507	3.551	.557	2.883
.358	7.466	.408	5.670	.458	4.429	.508	3.536	.558	2.871
.359	7.423	.409	5.641	.459	4.468	.509	3.521	.559	2.860
0.360	7.380	0.410	5.612	0.460	4.388	0.510	3.505	0.560	2.848
.361	7.337	.411	5.583	.461	4.367	.511	3.490	.561	2.837
.362	7.294	.412	5.564	.462	4.347	.512	3.475	.562	2.826
.363	7.253	.413	5.525	.463	4.326	.513	3.460	.563	2.814
.364	7.211	.414	5.497	.464	4.306	.514	3.446	.564	2.803
0.365	7.170	0.415	5.469	0.465	4.287	0.515	3.431	0.565	2.792
.366	7.129	.416	5.441	.466	4.267	.516	3.416	.566	2.780
.367	7.088	.417	5.413	.467	4.247	.517	3.402	.567	2.770
.368	7.048	.418	5.386	.468	4.227	.518	3.387	.568	2.759
.369	7.008	.419	5.359	.469	4.208	.519	3.373	.569	2.748
0.370	6.968	0.420	5.332	0.470	4.189	0.520	3.359	0.570	2.737
.371	6.929	.421	5.305	.471	4.169	.521	3.344	.571	2.726
.372	6.890	.422	5.278	.472	4.150	.522	3.330	.572	2.716
.373	6.851	.423	5.252	.473	4.131	.523	3.316	.573	2.705
.374	6.812	.424	5.225	.474	4.112	.524	3.302	.574	2.694
0.375	6.775	0.425	5.199	0.475	4.094	0.525	3.289	0.575	2.684
.376	6.737	.426	5.173	.476	4.075	.526	3.275	.576	2.673
.377	6.699	.427	5.147	.477	4.057	.527	3.261	.577	2.663
.378	6.662	.428	5.121	.478	4.038	.528	3.247	.578	2.652
.379	6.625	.429	5.096	.479	4.020	.529	3.244	.579	2.642
0.380	6.589	0.430	5.071	0.480	4.002	0.530	3.220	0.580	2.631
.381	6.552	.431	5.046	.481	3.984	.531	3.207	.581	2.621
.382	6.516	.432	5.021	.482	3.966	.532	3.193	.582	2.611
.383	6.480	.433	4.996	.483	3.948	.533	3.180	.583	2.601
.384	6.445	.434	4.971	.484	3.930	.534	3.167	.584	2.591
0.385	6.410	0.435	4.947	0.485	3.913	0.535	3.154	0.585	2.581
.386	6.375	.436	4.923	.486	3.895	.536	3.141	.586	2.571
.387	6.340	.437	4.899	.487	3.878	.537	3.128	.587	2.561
.388	6.306	.438	4.875	.488	3.860	.538	3.115	.588	2.551
.389	6.272	.439	4.851	.489	3.843	.539	3.102	.589	2.541
0.390	6.238	0.440	4.828	0.490	3.826	0.540	3.089	0.590	2.531
.391	6.204	.441	4.804	.491	3.809	.541	3.077	.591	2.521
.392	6.171	.442	4.781	.492	3.792	.542	3.064	.592	2.511
.393	6.134	.443	4.758	.493	3.775	.543	3.052	.593	2.502
.394	6.105	.444	4.735	.494	3.759	.544	3.039	.594	2.492
0.395	6.072	0.445	4.712	0.495	3.742	0.545	3.027	0.595	2.483
.396	6.040	.446	4.689	.496	3.726	.546	3.014	.596	2.474
.397	6.008	.447	4.667	.497	3.709	.547	3.002	.597	2.464
.398	5.976	.448	4.645	.498	3.693	.548	2.990	.598	2.455
.399	5.944	.449	4.622	.499	3.677	.549	2.978	.599	2.445
0.400	5.913	0.450	4.600	0.500	3.661	0.550	2.965	0.600	2.436

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
0.600	2.436	0.650	2.024	0.700	1.696	0.750	1.431	0.800	1.214
.601	2.427	.651	2.016	.701	1.690	.751	1.426	.802	1.206
.602	2.418	.652	2.019	.702	1.684	.752	1.422	.804	1.198
.603	2.408	.653	2.002	.703	1.678	.753	1.417	.806	1.190
.604	2.399	.654	1.995	.704	1.672	.754	1.412	.808	1.183
0.605	2.390	0.655	1.988	0.705	1.667	0.755	1.408	0.810	1.175
.606	2.381	.656	1.980	.706	1.661	.756	1.403	.812	1.168
.607	2.372	.657	1.973	.707	1.656	.757	1.398	.814	1.160
.608	2.363	.658	1.966	.708	1.650	.758	1.394	.816	1.153
.609	2.354	.659	1.959	.709	1.644	.759	1.389	.818	1.145
0.610	2.346	0.660	1.952	0.710	1.639	0.760	1.384	0.820	1.138
.611	2.337	.661	1.945	.711	1.633	.761	1.380	.822	1.130
.612	2.328	.662	1.938	.712	1.627	.762	1.375	.824	1.123
.613	2.319	.663	9.931	.713	1.622	.763	1.371	.826	1.116
.614	2.310	.664	1.924	.714	1.616	.764	1.366	.828	1.109
0.615	2.302	0.665	1.918	0.715	1.611	0.765	1.362	0.830	1.102
.616	2.293	.666	1.911	.716	1.605	.766	1.357	.832	1.095
.617	2.285	.667	1.904	.717	1.600	.767	1.353	.834	1.088
.618	2.276	.668	1.897	.718	1.594	.768	1.348	.836	1.081
.619	2.268	.669	1.890	.719	1.589	.769	1.344	.838	1.074
0.620	2.259	0.670	1.884	0.720	1.584	0.770	1.339	0.840	1.067
.621	2.251	.671	1.877	.721	1.578	.771	1.335	.842	1.060
.622	2.242	.672	1.870	.722	1.573	.772	1.330	.844	1.054
.623	2.234	.673	1.864	.723	1.567	.773	1.326	.846	1.047
.624	2.226	.674	1.857	.724	1.562	.774	1.322	.848	1.040
0.625	2.218	0.675	1.851	0.725	1.557	0.775	1.318	0.850	1.034
.626	2.209	.676	1.844	.726	1.552	.776	1.313	.852	1.027
.627	2.201	.677	1.838	.727	1.546	.777	1.309	.854	1.020
.628	2.193	.678	1.831	.728	1.541	.778	1.304	.856	1.014
.629	2.185	.679	1.825	.729	1.536	.779	1.300	.858	1.007
0.630	2.177	0.680	1.819	0.730	1.531	0.780	1.296	0.860	1.001
.631	2.169	.681	1.812	.731	1.526	.781	1.292	.862	.9947
.632	2.161	.682	1.806	.732	1.520	.782	1.287	.864	.9884
.633	2.153	.683	1.799	.733	1.515	.783	1.283	.866	.9821
.634	2.145	.684	1.792	.734	1.510	.784	1.279	.868	.9759
0.635	2.137	0.685	1.787	0.735	1.505	0.785	1.275	0.870	.9698
.636	2.130	.686	1.781	.736	1.500	.786	1.271	.872	.9636
.637	2.122	.687	1.774	.737	1.495	.787	1.266	.874	.9575
.638	2.114	.688	1.768	.738	1.490	.788	1.262	.876	.9514
.639	2.106	.689	1.762	.739	1.485	.789	1.258	.878	.9454
0.640	2.099	0.690	1.756	0.740	1.480	0.790	1.254	0.880	.9394
.641	2.091	.691	1.750	.741	1.475	.791	1.250	.882	.9334
.642	2.083	.692	1.744	.742	1.470	.792	1.246	.884	.9276
.643	2.076	.693	1.738	.743	1.465	.793	1.242	.886	.9217
.644	2.068	.694	1.732	.744	1.460	.794	1.238	.888	.9159
0.645	2.061	0.695	1.726	0.745	1.455	0.795	1.234	0.890	.9101
.646	2.053	.696	1.720	.746	1.450	.796	1.230	.892	.9043
.647	2.046	.697	1.714	.747	1.446	.797	1.226	.894	.8986
.648	2.038	.698	1.708	.748	1.441	.798	1.222	.896	.8930
.649	2.031	.699	1.702	.749	1.436	.799	1.218	.898	.8873
0.650	2.024	0.700	1.696	0.750	1.431	0.800	1.214	0.900	.8817

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
0.900	.8817	1.000	.6421	1.100	.4627	1.200	.3240	1.300	.2135
.902	.8762	1.002	.6380	1.102	.4596	1.202	.3215	1.302	.2116
.904	.8706	1.004	.6339	1.104	.4565	1.204	.3191	1.304	.2096
.906	.8651	1.006	.6299	1.106	.4534	1.206	.3166	1.306	.2076
.908	.8597	1.008	.6258	1.108	.4503	1.208	.3142	1.308	.2057
0.910	.8543	1.010	.6218	1.110	.4472	1.210	.3118	1.310	.2037
.912	.8489	1.012	.6178	1.112	.4442	1.212	.3094	1.312	.2018
.914	.8435	1.014	.6139	1.114	.4412	1.214	.3070	1.314	.1998
.916	.8382	1.016	.6100	1.116	.4382	1.216	.3047	1.316	.1979
.918	.8329	1.018	.6060	1.118	.4352	1.218	.3023	1.318	.1960
0.920	.8277	1.020	.6022	1.120	.4321	1.220	.2999	1.320	.1941
.922	.8225	1.022	.5983	1.122	.4292	1.222	.2976	1.322	.1922
.924	.8173	1.024	.5944	1.124	.4262	1.224	.2953	1.324	.1903
.926	.8121	1.026	.5906	1.126	.4233	1.226	.2929	1.326	.1884
.928	.8070	1.028	.5868	1.128	.4204	1.228	.2906	1.328	.1665
0.930	.8019	1.030	.5830	1.130	.4175	1.230	.2883	1.330	.1847
.932	.7968	1.032	.5792	1.132	.4146	1.232	.2860	1.332	.1828
.934	.7818	1.039	.5755	1.134	.4117	1.234	.2837	1.334	.1809
.936	.7868	1.036	.5718	1.136	.4088	1.236	.2815	1.336	.1790
.938	.7819	1.038	.5681	1.138	.4060	1.238	.2792	1.338	.1772
0.940	.7764	1.040	.5644	1.140	.4032	1.240	.2769	1.340	.1754
.942	.7720	1.042	.5607	1.142	.4003	1.242	.2747	1.342	.1735
.944	.7672	1.044	.5571	1.144	.3975	1.244	.2725	1.344	.1717
.946	.7623	1.046	.5535	1.146	.3947	1.246	.2702	1.346	.1699
.948	.7575	1.048	.5499	1.148	.3919	1.248	.2680	1.348	.1681
0.950	.7527	1.050	.5463	1.150	.3891	1.250	.2658	1.350	.1663
.952	.7480	1.052	.5427	1.152	.3864	1.252	.2636	1.352	.1645
.954	.7433	1.054	.5392	1.154	.3836	1.254	.2614	1.354	.1627
.956	.7386	1.056	.5357	1.156	.3809	1.256	.2592	1.356	.1609
.958	.7339	1.058	.5322	1.158	.3782	1.258	.2571	1.358	.1591
0.960	.7293	1.060	.5287	1.160	.3755	1.260	.2549	1.360	.1574
.962	.7247	1.062	.5252	1.162	.3728	1.262	.2528	1.362	.1556
.964	.7201	1.064	.5218	1.164	.3701	1.264	.2506	1.364	.1538
.966	.7155	1.066	.5183	1.166	.3675	1.266	.2485	1.366	.1520
.968	.7110	1.068	.5149	1.168	.3648	1.268	.2464	1.368	.1503
0.970	.7065	1.070	.5115	1.170	.3621	1.270	.2443	1.370	.1485
.972	.7020	1.072	.5082	1.172	.3595	1.272	.2422	1.372	.1468
.974	.6976	1.074	.5048	1.174	.3569	1.274	.2400	1.374	.1451
.976	.6932	1.076	.5014	1.176	.3543	1.276	.2380	1.376	.1434
.978	.6888	1.078	.4981	1.178	.3517	1.278	.2359	1.378	.1417
0.980	.6844	1.080	.4948	1.180	.3492	1.280	.2338	1.380	.1399
.982	.6800	1.082	.4915	1.182	.3466	1.282	.2317	1.382	.1382
.984	.6757	1.084	.4883	1.184	.3440	1.284	.2297	1.384	.1365
.986	.6714	1.086	.4850	1.186	.3415	1.286	.2284	1.386	.1349
.988	.6672	1.088	.4818	1.188	.3389	1.288	.2256	1.388	.1332
0.990	.6630	1.090	.4785	1.190	.3364	1.290	.2236	1.390	.1315
.992	.6587	1.092	.4753	1.192	.3339	1.292	.2216	1.392	.1298
.994	.6545	1.094	.4722	1.194	.3314	1.294	.2195	1.394	.1282
.996	.6504	1.096	.4690	1.196	.3289	1.296	.2175	1.396	.1265
.998	.6462	1.098	.4658	1.198	.3264	1.298	.2155	1.398	.1248
1.000	.6421	1.100	.4627	1.200	.3240	1.300	.2135	1.400	.1232

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
1.400	.1232	1.50	.04728	2.00	-.2289	2.50	- .5354	3.00	- 2.338
1.402	.1216	1.51	.04033	2.01	-.2337	2.51	- .5446	3.01	- 2.510
1.404	.1199	1.52	.03342	2.02	-.2387	2.52	- .5540	3.02	- 2.710
1.406	.1183	1.53	.02667	2.03	-.2436	2.53	- .5636	3.03	- 2.945
1.418	.1166	1.54	.02000	2.04	-.2485	2.54	- .5735	3.04	- 3.227
1.410	.1150	1.55	.01342	2.05	-.2535	2.55	- .5836	3.05	- 3.570
1.412	.1134	1.56	.00692	2.06	-.2584	2.56	- .5941	3.06	- 3.996
1.414	.1118	1.57	.00051	2.07	-.2634	2.57	- .6049	3.07	- 4.540
1.416	.1102	1.58	-.00582	2.08	-.2684	2.58	- .6160	3.08	- 5.347
1.418	.1086	1.59	-.01208	2.09	-.2734	2.59	- .6275	3.09	- 6.252
1.420	.1070	1.60	-.01825	2.10	-.2785	2.60	- .6393	3.10	- 7.751
1.422	.1054	1.61	-.02435	2.11	-.2836	2.61	- .6515	3.11	- 10.176
1.424	.1038	1.62	-.03037	2.12	-.2887	2.62	- .6642	3.12	- 14.84
1.426	.1022	1.63	-.03638	2.13	-.2938	2.63	- .6772	3.13	-.27.57
1.428	.1007	1.64	-.04226	2.14	-.2990	2.64	- .6907	3.14	-200.4
1.430	.09909	1.65	-.04812	2.15	-.3042	2.65	- .7048	3.15	37.74
1.432	.09755	1.66	-.05386	2.16	-.3094	2.66	- .7193	3.16	17.19
1.434	.09600	1.67	-.05958	2.17	-.3147	2.67	- .7344	3.17	11.10
1.436	.09444	1.68	-.06523	2.18	-.3200	2.68	- .7501	3.18	8.182
1.438	.09289	1.69	-.07089	2.19	-.3254	2.69	- .7664	3.19	6.473
1.440	.09132	1.70	-.07641	2.20	-.3309	2.70	- .7835	3.20	5.343
1.442	.08981	1.71	-.08193	2.21	-.3363	2.71	- .8012	3.21	4.545
1.444	.08828	1.72	-.08738	2.22	-.3418	2.72	- .8197	3.22	3.953
1.446	.08676	1.73	-.09277	2.23	-.3474	2.73	- .8391	3.23	3.492
1.448	.08523	1.74	-.09816	2.24	-.3531	2.74	- .8594	3.24	3.126
1.450	.08372	1.75	-.1035	2.25	-.3588	2.75	- .8807	3.25	2.827
1.452	.08220	1.76	-.1088	2.26	-.3646	2.76	- .9029	3.26	2.579
1.454	.08069	1.77	-.1141	2.27	-.3704	2.77	- .9264	3.27	2.369
1.456	.07919	1.78	-.1193	2.28	-.3764	2.78	- .9510	3.28	2.189
1.458	.07770	1.79	-.1245	2.29	-.3824	2.79	- .9771	3.29	2.033
1.460	.07623	1.80	-.1296	2.30	-.3885	2.80	-1.005	3.30	1.897
1.462	.07471	1.81	-.1347	2.31	-.3946	2.81	-1.034	3.31	1.777
1.464	.07322	1.82	-.1398	2.32	-.4009	2.82	-1.064	3.32	1.670
1.466	.07175	1.83	-.1449	2.33	-.4073	2.83	-1.097	3.33	1.575
1.468	.07027	1.84	-.1499	2.34	-.4173	2.84	-1.132	3.34	1.489
1.470	.06878	1.85	-.1550	2.35	-.4203	2.85	-1.169	3.35	1.412
1.472	.06734	1.86	-.1599	2.36	-.4269	2.86	-1.209	3.36	1.341
1.474	.06587	1.87	-.1649	2.37	-.4338	2.87	-1.251	3.37	1.277
1.476	.06442	1.88	-.1699	2.38	-.4406	2.88	-1.297	3.38	1.217
1.478	.06297	1.89	-.1749	2.39	-.4477	2.89	-1.346	3.39	1.163
1.480	.06149	1.90	-.1798	2.40	-.4549	2.90	-1.399	3.40	1.113
1.482	.06007	1.91	-.1847	2.41	-.4622	2.91	-1.457	3.41	1.066
1.484	.05864	1.92	-.1896	2.42	-.4696	2.92	-1.520	3.42	1.023
1.486	.05720	1.93	-.1946	2.43	-.4772	2.93	-1.589	3.43	.9827
1.488	.05577	1.94	-.1994	2.44	-.4850	2.94	-1.664	3.44	.9450
1.490	.05436	1.95	-.2044	2.45	-.4929	2.95	-1.747	3.45	.9098
1.492	.05292	1.96	-.2092	2.46	-.5011	2.96	-1.840	3.46	.8768
1.494	.05150	1.97	-.2142	2.47	-.5093	2.97	-1.943	3.47	.8457
1.496	.05009	1.98	-.2190	2.48	-.5175	2.98	-2.058	3.48	.8164
1.498	.04868	1.99	-.2239	2.49	-.5265	2.99	-2.189	3.49	.7888
1.500	.04728	2.00	-.2289	2.50	-.5354	3.00	-2.338	3.50	.7627

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
3.50	.7627	4.00	.2159	4.50	.04792	5.00	-.05916	6.00	- .5727
3.51	.7380	4.01	.2111	4.51	.04550	5.02	-.06328	6.02	- .6165
3.52	.7146	4.02	.2063	4.52	.04310	5.04	-.06743	6.04	- .6673
3.53	.6923	4.03	.2016	4.53	.04072	5.06	-.07160	6.06	- .7270
3.54	.6711	4.04	.1971	4.54	.03835	5.08	-.07581	6.08	- .7983
3.55	.6509	4.05	.1926	4.55	.03601	5.10	-.08005	6.10	- .8849
3.56	.6317	4.06	.1882	4.56	.03368	5.12	-.08433	6.12	- .9924
3.57	.6133	4.07	.1838	4.57	.03137	5.14	-.08866	6.14	- 1.130
3.58	.5958	4.08	.1796	4.58	.02908	5.16	-.09304	6.16	- 1.311
3.59	.5790	4.09	.1754	4.59	.02680	5.18	-.09748	6.18	- 1.562
3.60	.5629	4.10	.1713	4.60	.02453	5.20	-.1020	6.20	- 1.934
3.61	.5474	4.11	.1673	4.61	.02229	5.22	-.1066	6.22	- 2.541
3.62	.5326	4.12	.1634	4.62	.02005	5.24	-.1112	6.24	- 3.708
3.63	.5184	4.13	.1595	4.63	.01783	5.26	-.1159	6.26	- 6.887
3.64	.5047	4.14	.1556	4.64	.01563	5.28	-.1208	6.28	-49.98
3.65	.4916	4.15	.1519	4.65	.01343	5.30	-.1257	6.30	9.439
3.66	.4789	4.16	.1482	4.66	.01125	5.32	-.1307	6.32	4.295
3.67	.4667	4.17	.1445	4.67	.009082	5.34	-.1358	6.34	2.773
3.68	.4549	4.18	.1409	4.68	.006923	5.36	-.1411	6.36	2.043
3.69	.4436	4.19	.1374	4.69	.004874	5.38	-.1465	6.38	1.614
3.70	.4326	4.20	.1339	4.70	.002636	5.40	-.1521	6.40	1.332
3.71	.4219	4.21	.1305	4.71	.000507	5.42	-.1578	6.42	1.131
3.72	.4117	4.22	.1271	4.72	-.001609	5.44	-.1637	6.44	.9820
3.73	.4017	4.23	.1238	4.73	-.003724	5.46	-.1698	6.46	.8663
3.74	.3921	4.24	.1205	4.74	-.005827	5.48	-.1761	6.48	.7739
3.75	.3828	4.25	.1173	4.75	-.007922	5.50	-.1826	6.50	.6984
3.76	.3738	4.26	.1141	4.76	-.01001	5.52	-.1893	6.52	.6355
3.77	.3650	4.27	.1109	4.77	-.01209	5.54	-.1964	6.54	.5822
3.78	.3565	4.28	.1078	4.78	-.01417	5.56	-.2037	6.56	.5365
3.79	.3482	4.29	.1048	4.79	-.01624	5.58	-.2114	6.58	.4969
3.80	.3401	4.30	.1017	4.80	-.01830	5.60	-.2194	6.60	.4621
3.81	.3323	4.31	.09875	4.81	-.02036	5.62	-.2248	6.62	.4314
3.82	.3247	4.32	.09580	4.82	-.02241	5.64	-.2366	6.64	.4040
3.83	.3173	4.33	.09288	4.83	-.02446	5.66	-.2458	6.66	.3794
3.84	.3101	4.34	.09000	4.84	-.02651	5.68	-.2556	6.68	.3572
3.85	.303[illegible]	4.35	.08716	4.85	-.02855	5.70	-.2659	6.70	.3371
3.86	.2963	4.36	.08434	4.86	-.03059	5.72	-.2769	6.72	.3187
3.87	.2896	4.37	.08156	4.87	-.03263	5.74	-.2886	6.74	.3019
3.88	.2831	4.38	.07881	4.88	-.03467	5.76	-.3010	6.76	.2864
3.89	.2768	4.39	.07609	4.89	-.03671	5.78	-.4143	6.78	.2720
3.90	.2707	4.40	.07340	4.90	-.03874	5.80	-.3286	6.80	.2588
3.91	.2646	4.41	.07074	4.91	-.04078	5.82	-.3440	6.82	.2463
3.92	.2586	4.42	.06810	4.92	-.04282	5.84	-.3607	6.84	.2348
3.93	.2529	4.43	.06549	4.93	-.04485	5.86	-.3789	6.86	.2240
3.94	.2473	4.44	.06291	4.94	-.04689	5.88	-.3987	6.88	.2139
3.95	.2418	4.45	.06036	4.95	-.04893	5.90	-.4204	6.90	.2044
3.96	.2364	4.46	.05782	4.96	-.05097	5.92	-.4445	6.92	.1954
3.97	.2311	4.47	.05531	4.97	-.05301	5.94	-.4711	6.94	.1869
3.98	.2260	4.48	.05283	4.98	-.05506	5.96	-.5010	6.96	.1788
3.99	.2209	4.49	.05036	4.99	-.05711	5.98	-.5345	6.98	.1712
4.00	.2159	4.50	.04792	5.00	-.05916	6.00	-.5727	7.00	.1639

$\frac{\cot x}{x}$ (continued)

x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$	x	$\frac{\cot x}{x}$
7.00	.1639	8.00	-.01838	9.00	- .2456	10.0	.1542	15.0	- .07788	15.0	- .07788
7.02	.1570	8.02	-.02089	9.02	- .2588	10.1	.1236	15.1	- .09517	15.1	- .09517
7.04	.1504	8.04	-.02341	9.04	- .2732	10.2	.1001	15.2	- .1182	15.2	- .1182
7.06	.1441	8.06	-.02593	9.06	- .2890	10.3	.08104	15.3	- .1512	15.3	- .1512
7.08	.1380	8.08	-.02846	9.08	- .3067	10.4	.06516	15.4	- .2041	15.4	- .2041
7.10	.1323	8.10	-.03100	9.10	- .3264	10.5	.05148	15.5	- .3057	15.5	- .3057
7.12	.1267	8.12	-.03356	9.12	- .3486	10.6	.03939	15.6	- .5914	15.6	- .5914
7.14	.1214	8.14	-.03613	9.14	- .3738	10.7	.02846	15.7	-7.998	15.7	-7.998
7.16	.1162	8.16	-.03872	9.16	- .4026	10.8	.01834	15.8	.6857	15.8	.6857
7.18	.1112	8.18	-.04133	9.18	- .4361	10.9	.008795	15.9	.3235	15.9	.3235
7.20	.1065	8.20	-.04397	9.20	- .4754	11.0	- .000403	16.0	.2079	16.0	.2079
7.22	.1018	8.22	-.04663	9.22	- .5222	11.1	- .009442	16.1	.1502	16.1	.1502
7.24	.09735	8.24	-.04932	9.24	- .5790	11.2	- .01851	16.2	.1152	16.2	.1152
7.26	.09302	8.26	-.05205	9.26	- .6494	11.3	- .02780	16.3	.09122	16.3	.09122
7.28	.08882	8.28	-.05481	9.28	- .7391	11.4	- .03755	16.4	.07357	16.4	.07357
7.30	.08474	8.30	-.06761	9.30	- .8573	11.5	- .04801	16.5	.05981	16.5	.05981
7.32	.08077	8.32	-.06045	9.32	- 1.020	11.6	- .05954	16.6	.04859	16.6	.04859
7.34	.07692	8.34	-.06334	9.34	- 1.260	11.7	- .07264	16.7	.03913	16.7	.03913
7.36	.07316	8.36	-.06628	9.36	- 1.647	11.8	- .08803	16.8	.03089	16.8	.03089
7.38	.06951	8.38	-.06928	9.38	- 2.379	11.9	- .1067	16.9	.02355	16.9	.02355
7.40	.06594	8.40	-.07237	9.40	- 4.292	12.0	- .1311	17.0	.01684	17.0	.01684
7.42	.06246	8.42	-.07783	9.42	-22.23	12.1	- .1642	17.1	.01057	17.1	.01057
7.44	.05906	8.44	-.07865	9.44	6.960	12.2	- .2136	17.2	.004590	17.2	.004590
7.46	.05572	8.46	-.08192	9.46	3.000	12.3	- .2980	17.3	- .001228	17.3	- .001228
7.48	.05247	8.48	-.08526	9.48	1.908	12.4	- .4802	17.4	- .007002	17.4	- .007002
7.50	.04927	8.50	-.08870	9.50	1.397	12.5	-1.204	17.5	- .01285	17.5	- .01285
7.52	.04614	8.52	-.09223	9.52	1.100	12.6	2.359	17.6	- .01891	17.6	- .01891
7.54	.04307	8.54	-.09586	9.54	.9057	12.7	.5857	17.7	- .02531	17.7	- .02531
7.56	.04005	8.56	-.09961	9.56	.7688	12.8	.3283	17.8	- .03226	17.8	- .03226
7.58	.03708	8.58	-.1035	9.58	.6671	12.9	.2237	17.9	- .03999	17.9	- .03999
7.60	.03416	8.60	-.1075	9.60	.5884	13.0	.1661	18.0	- .04885	18.0	- .04885
7.62	.03128	8.62	-.1116	9.62	.5257	13.1	.1292	18.1	- .05936	18.1	- .05936
7.64	.02844	8.64	-.1159	9.64	.4745	13.2	.1031	18.2	- .07234	18.2	- .07234
7.66	.02565	8.66	-.1203	9.66	.4319	13.3	.08341	18.3	- .08922	18.3	- .08922
7.68	.02288	8.68	-.1250	9.68	.3959	13.4	.06775	18.4	- .1126	18.4	- .1126
7.70	.02016	8.70	-.1298	9.70	.3651	13.5	.05483	18.5	- .1483	18.5	- .1483
7.72	.01746	8.72	-.1348	9.72	.3383	13.6	.04379	18.6	- .2110	18.6	- .2110
7.74	.01479	8.74	-.1401	9.74	.3148	13.7	.03411	18.7	- .3549	18.7	- .3549
7.76	.01215	8.76	-.1456	9.76	.2941	13.8	.02540	18.8	-1.072	18.8	-1.072
7.78	.009526	8.78	-.1514	9.78	.2756	13.9	.01739	18.9	1.048	18.9	1.048
7.80	.006927	8.80	-.1576	9.80	.2591	14.0	.009859	19.0	.3472	19.0	.3472
7.82	.004345	8.82	-.1640	9.82	.2441	14.1	.002637	19.1	.2047	19.1	.2047
7.84	.001783	8.84	-.1709	9.84	.2305	14.2	- .004430	19.2	.1425	19.2	.1425
7.86	-.000766	8.86	-.1781	9.86	.2181	14.3	- .01149	19.3	.1071	19.3	.1071
7.88	-.003326	8.88	-.1858	9.88	.2068	14.4	- .01868	19.4	.08399	19.4	.08399
7.90	-.005829	8.90	-.1941	9.90	.1963	14.5	- .02618	19.5	.06740	19.5	.06740
7.92	-.008348	8.92	-.2029	9.92	.1866	14.6	- .03418	19.6	.05472	19.6	.05472
7.94	-.01086	8.94	-.2124	9.94	.1777	14.7	- .04292	19.7	.04455	19.7	.04455
7.96	-.01337	8.96	-.2226	9.96	.1693	14.8	- .05275	19.8	.03608	19.8	.03608
7.98	-.01588	8.98	.2336	9.98	.1615	14.9	- .06415	19.9	.02880	19.9	.02880
8.00	-.01838	9.00	-.2456	10.00	.1542	15.0	- .07788	20.0	.02235	20.0	.02235